5.1 The Derivative of the Natural Exponential

Problems take from the Nelson Text – Pg. 232 – 233

2. Differentiate each of the following:

   a. $y = e^{3x}$
   b. $s = e^{3t-5}$
   c. $y = 2e^{10t}$
   d. $y = e^{-3x}$
   e. $y = e^{5-6x+x^2}$
   f. $y = e^{\sqrt{x}}$

3. Determine the derivative of each of the following:

   a. $y = 2e^{x^3}$
   b. $y = xe^{3x}$
   c. $f(x) = \dfrac{e^{-x^3}}{x}$
   d. $f(x) = \sqrt{x}e^{x}$
   e. $h(t) = et^2 + 3e^{-t}$
   f. $g(t) = \dfrac{e^{2t}}{1 + e^{2t}}$

4. a. If $f(x) = \frac{1}{3}(e^{3x} + e^{-3x})$, calculate $f'(1)$.
   b. If $f(x) = e^{-\left(\frac{1}{x+1}\right)}$, calculate $f'(0)$.
   c. If $h(z) = z^2(1 + e^{-z})$, calculate $h'(-1)$.

6. Determine the equation of the tangent to the curve $y = e^{-x}$ at the point where $x = -1$. Graph the original curve and the tangent.

9. If $y = \frac{5}{2}(e^{\frac{x}{5}} + e^{-\frac{x}{5}})$, prove that $y'' = \frac{y}{25}$.

11. Determine the first and second derivatives of each function.

   a. $y = -3e^{x}$
   b. $y = xe^{2x}$
   c. $y = e^{x}(4 - x)$

(*over*)

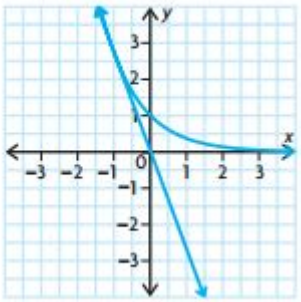
12. The number, N, of bacteria in a culture at time t, in hours, is $N(t) = 1000\left[30 + e^{-\frac{t}{30}}\right]$
    a. What is the initial number of bacteria in the culture?
    b. Determine the rate of change in the number of bacteria at time t.
    c. How fast is the number of bacteria changing when $t = 20$?
    d. Determine the largest number of bacteria in the culture during the interval $0 \leq t \leq 50$.
    e. What is happening to the number of bacteria in the culture as time passes?

13. The distance s, in metres, fallen by a skydiver t seconds after jumping (and before the parachute opens) is $s = 160\left(\frac{1}{4}t - 1 + e^{-\frac{t}{4}}\right)$.
    a. Determine the velocity, v, at time t.
    b. Show that acceleration is given by $a = 10 - \frac{1}{4}v$.
    c. Determine $v_T = \lim_{t\to\infty} v$. This is the "terminal" velocity, the constant velocity attained when the air resistance balances the force of gravity.
    d. At what time is the velocity 95% of the terminal velocity? How far has the skydiver fallen at that time?

Answers to Selected Problems

3. **a.** $6x^2e^{x^3}$
b. $e^{3x}(3x + 1)$
c. $\dfrac{-3x^2e^{-x^3}(x) - e^{-x^3}}{x^2}$
d. $\sqrt{x}e^x + e^x\left(\dfrac{1}{2\sqrt{x}}\right)$
e. $2te^{t^2} - 3e^{-t}$
f. $\dfrac{2e^{2t}}{(1 + e^{2t})^2}$

6. $ex + y = 0$



12. **a.** 31 000
b. $-\dfrac{100}{3}e^{-\frac{t}{30}}$
c. -17 bacteria/h
d. 31 000 at time $t = 0$
e. The number of bacteria is constantly decreasing as time passes.