5.3 Optimization with Exponential Functions

These problems taken from the Nelson Text – Pg. 245 – 247

4. The net monthly profit, in dollars, from the sale of a certain item is given by the formula $P(x) = 10^6[1 + (x - 1)e^{-0.001x}]$, where x is the number of items sold.
   a. Determine the number of items that yield the maximum profit. At full capacity, the factory can produce 2000 items per month.
   b. Repeat part a., assuming that, at most, 500 items can be produced per month.
6. A rumour spreads through a population in such a way that t hours after the rumour starts, the percent of people involved in passing it on is given by $P(t) = 100\left(e^{-t} - e^{-4t}\right)$. What is the highest percent of people involved in spreading the rumour within the first 3 h? When does this occur?
8. A colony of bacteria in a culture grows at a rate given by $N(t) = 2^{\frac{t}{5}}$, where N is the number of bacteria t minutes from the beginning. The colony is allowed to grow for 60 min, at which time a drug is introduced to kill the bacteria. The number of bacteria killed is given by $K(t) = e^{\frac{t}{3}}$, where K bacteria are killed at time t minutes.
   a. Determine the maximum number of bacteria present and the time at which this occurs.
   b. Determine the time at which the bacteria colony is obliterated.

12. Find the maximum and minimum values of each function. Graph each function.

    c. $y = 2xe^{2x}$ d. $y = 3xe^{-x} + x$

13. The profit function of a commodity is $P(x) = xe^{-0.5x^2}$, where $x > 0$. Find the maximum value of the function if x is measured in hundreds of units and P is measured in thousands of dollars.

(*over*)

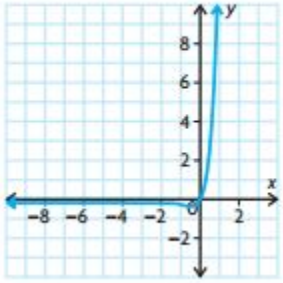
Answers to Selected Problems

4. **a.** 1001 items
b. 500 items

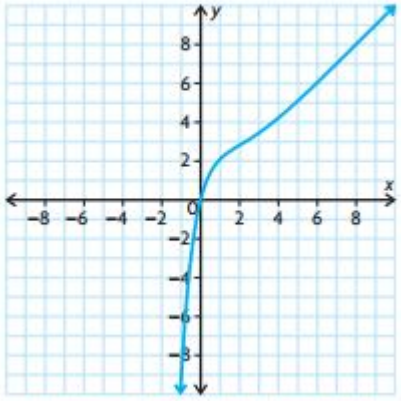
6. 47.2%; 0.462 h

8. **a.** 478 158; 38.2 min after the drug was introduced
b. 42.72 min after the drug was introduced

12. **c.** min: $-e^{-1} \doteq -0.37$, no max



d. no max or min



13. about 0.61