5.4 The Derivatives of Sine and Cosine

These problems are taken from the Nelson text: Pg. 256 – 257

1. Determine $\frac{dy}{dx}$ for each of the following:

a. $y = \sin 2x$

b. $y = 2\cos 3x$

c. $y = \sin(x^3 - 2x + 4)$

d. $y = 2\cos(-4x)$

e. $y = \sin 3x - \cos 4x$

f. $y = 2^x + 2\sin x - 2\cos x$

g. $y = \sin(e^x)$

h. $y = 3\sin(3x + 2\pi)$

i. $y = x^2 + \cos x + \sin\frac{\pi}{4}$

j. $y = \sin\frac{1}{x}$

2. Differentiate the following functions:

a. $y = 2\sin x\cos x$

b. $y = \frac{\cos 2x}{x}$

c. $y = \cos(\sin 2x)$

d. $y = \frac{\sin x}{1 + \cos x}$

e. $y = e^x(\cos x + \sin x)$

f. $y = 2x^3\sin x - 3x\cos x$

3. Determine an equation for the tangent at the point with the given x-coordinate for each of the following functions:

a. $f(x) = \sin x, x = \frac{\pi}{3}$

b. $f(x) = x + \sin x, x = 0$

d. $f(x) = \sin 2x + \cos x, x = \frac{\pi}{2}$

e. $f(x) = \cos\left(2x + \frac{\pi}{3}\right), x = \frac{\pi}{4}$

(*over*)

5. Differentiate each function.

a. $v(t) = \sin^2(\sqrt{t})$

b. $v(t) = \sqrt{1 + \cos t + \sin^2 t}$

c. $h(x) = \sin x \sin 2x \sin 3x$

d. $m(x) = (x^2 + \cos^2 x)^3$

6. Determine the absolute extreme values of each function on the given interval. (Verify your results with graphing technology.)

a. $y = \cos x + \sin x,\ 0 \leq x \leq 2\pi$

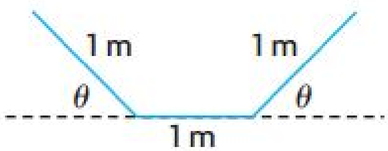
c. $y = \sin x - \cos x,\ x \in [0, 2\pi]$

7. A particle moves along a line so that, at time t, its position is $s(t) = 8 \sin 2t$.

a. For what values of t does the particle change direction?

b. What is the particle's maximum velocity?

12. An irrigation channel is constructed by bending a sheet of metal that is 3 m wide, as shown in the diagram. What angle θ will maximize the cross-sectional area (and thus the capacity) of the channel?



Answers to Selected Problems

1. **a.** $2 \cos 2x$
b. $-6 \sin 3x$
c. $(3x^2 - 2)(\cos(x^3 - 2x + 4))$
d. $8 \sin(-4x)$
e. $3 \cos(3x) + 4 \sin(4x)$
f. $2^x(\ln 2) + 2 \cos x + 2 \sin x$
g. $e^x \cos(e^x)$
h. $9 \cos(3x + 2\pi)$
i. $2x - \sin x$
j. $-\frac{1}{x^2} \cos\left(\frac{1}{x}\right)$

2. **a.** $2 \cos(2x)$
b. $-\frac{2 \sin 2x}{x} - \frac{\cos 2x}{x^2}$
c. $-\sin(\sin 2x) \times 2 \cos 2x$
d. $\frac{1}{1 + \cos x}$
e. $e^x(2 \cos x)$
f. $2x^3 \cos x + 6x^2 \sin x + 3x \sin x - 3 \cos x$

3. **a.** $-x + 2y + \left(\frac{\pi}{3} - \sqrt{3}\right) = 0$
b. $-2x + y = 0$
d. $y = -3\left(x - \frac{\pi}{2}\right)$
e. $y + \frac{\sqrt{3}}{2} = -\left(x - \frac{\pi}{4}\right)$

5. **a.** $v'(t) = \frac{\sin(\sqrt{t}) \cos(\sqrt{t})}{\sqrt{t}}$
b. $v'(t) = \frac{-\sin t + 2(\sin t)(\cos t)}{2\sqrt{1 + \cos t + \sin^2 t}}$
c. $h'(x) = 3 \sin x \sin 2x \cos 3x + 2 \sin x \sin 3x \cos 2x + \sin 2x \sin 3x \cos x$
d. $m'(x) = 3(x^2 + \cos^2 x)^2 \times (2x - 2 \sin x \cos x)$

6. **a.** absolute max: $\sqrt{2}$, absolute min: $-\sqrt{2}$
b. absolute max: 2.26, absolute min: -5.14
c. absolute max: $\sqrt{2}$, absolute min: $-\sqrt{2}$
d. absolute max: 5, absolute min: -5

7. **a.** $t = \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k$ for positive integers k
b. 8

12. $\theta = \frac{\pi}{3}$