MCV4U

PRACTICE EXAM SOLUTIONS

MULTIPLE CHOICE

1	2	3	4	5	6	7	8	9	10
D	B	C	C	A	B	B	D	A	A

FULL SOLUTIONS.

11. Using the fact that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, determine $f'(x)$ for the function $f(x) = \tan(x)$.

$$f(x) = \tan(x)$$
$$= \frac{\sin(x)}{\cos(x)}$$
$$f'(x) = \frac{\cos(x)\cdot\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

12. Find the derivative of $f(x) = \sqrt{3x-2}$ from first principles.

$$f'(x) = \lim_{h\to 0}\left(\frac{\sqrt{3(x+h)-2} - \sqrt{3x-2}}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{\sqrt{3(x+h)-2} - \sqrt{3x-2}}{h} \cdot \frac{\sqrt{3(x+h)-2} + \sqrt{3x-2}}{\sqrt{3(x+h)-2} + \sqrt{3x-2}}\right)$$
$$= \lim_{h\to 0}\left(\frac{3(x+h) - 2 - (3x-2)}{h\left(\sqrt{3(x+h)-2} + \sqrt{3x-2}\right)}\right)$$
$$= \lim_{h\to 0}\left(\frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3(x+h)-2} + \sqrt{3x-2}\right)}\right)$$
$$= \frac{3}{2\sqrt{3x-2}}$$

13. Determine the first derivative of each of the following (simplify your answers as much as possible for **T** points):

a) $h(x) = \left(\frac{x-2}{1-x}\right)^2$ **K/2 T/1**

b) $f(x) = 3^{\sqrt{2x-3}}$ **K/2**

c) $g(x) = \cos^2(x) \cdot \ln(\sin(x))$ **K/3 T/1**

d) $y = 2x^3 \cdot e^{\sin(x^2)}$ **K/3 T/1**

a) $h(x) = \left(\frac{x-2}{1-x}\right)^2$

$$h'(x) = 2\left(\frac{x-2}{1-x}\right)\left(\frac{(1)(1-x) - (x-2)(-1)}{(1-x)^2}\right)$$

$$= 2\left(\frac{x-2}{1-x}\right)\left(\frac{-1}{(1-x)^2}\right)$$

$$= \frac{-2(x-2)}{(1-x)^3}$$

b) $f(x) = 3^{\sqrt{2x-3}}$

$$f'(x) = 3^{\sqrt{2x-3}} \cdot \ln(3) \cdot \frac{1}{\cancel{2}\sqrt{2x-3}} \cdot \cancel{2}$$

$$= 3^{\sqrt{2x-3}} \cdot \frac{\ln(3)}{\sqrt{2x-3}}$$

c) $g(x) = \cos^2 x \cdot \ln(\sin(x))$ $\qquad \cos^2 x = (\cos(x))^2$

$$g'(x) = 2\cos(x)(-\sin(x)) \cdot \ln(\sin(x)) + \cos^2 x \cdot \frac{1}{\sin(x)} \cdot \cos(x)$$

$$= -2\cos(x)\sin(x)\ln(\sin(x)) + \frac{\cos^3(x)}{\sin(x)}$$

d) $y = 2x^3 \cdot e^{\sin(x^2)}$

$y' = 6x^2 \cdot e^{\sin(x^2)} + 2x^3\left(e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x\right)$

$= 2x^2 e^{\sin(x^2)}\left(3 + 2x^2 \cos(x^2)\right)$

14. Consider the function $f(x) = x - e^{3x}$.
 a. Determine the absolute extrema of the function on the interval $-4 \le x \le 0$. **K/4**
 b. Do the absolute extrema change on the interval $-4 \le x \le -1$? Explain. If there is a new absolute extrema, determine it. **T/2**

a) $f'(x) = 1 - 3e^{3x}$ set to zero

$\Rightarrow 1 - 3e^{3x} = 0$

$\Rightarrow e^{3x} = \frac{1}{3}$

$\Rightarrow \ln(e^{3x}) = \ln\left(\frac{1}{3}\right)$

$\Rightarrow 3x = \ln\left(\frac{1}{3}\right)$

$\Rightarrow x = \frac{\ln\left(\frac{1}{3}\right)}{3} = -0.366$

Test $f(-4) = -4 - e^{-12} = -4$

$f(-0.366) = -0.366 - e^{3\left(\frac{\ln(\frac{1}{3})}{3}\right)} = -0.699$

$f(0) = 0 - e^0 = -1$

$\therefore$ max occurs at $x = -0.366$ and is -0.699

min occur at $x = -4$ and is -1

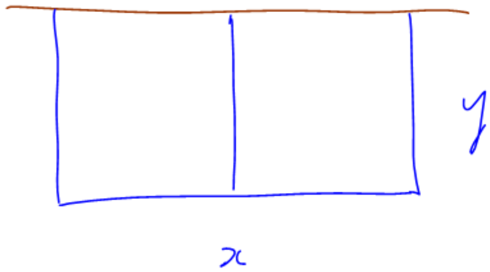
b) The C.V. is not in $[-4, -1]$

Test $f(-1) = -1 - e^{-3} = -1.05$ which is the new max

The min remains $f(-4) = -4$.

15. A farmer needs to fence off an area of 600 m², making two pens of equal size. He will use an existing wood fence on his farm for one side of the enclosed region and will construct the other pieces of the fence with wire fencing. Find the dimensions of the pens that minimize the length of the new wire fencing. (Be sure to **prove** your answer provides a minimum) **A/5**

Two possible drawings $\Rightarrow$ 2 possible answer $\Rightarrow$ either is "correct."



$A = xy$

$600 = xy$

$y = \frac{600}{x}$ (*)

$F = x + 3y$

$F(x) = x + \frac{1800}{x}$

$F'(x) = 1 - \frac{1800}{x^2}$

set to zero $\Rightarrow 1 - \frac{1800}{x^2} = 0$

$\Rightarrow 1 = \frac{1800}{x^2}$

$\Rightarrow x = \pm\sqrt{1800}$ ($-\sqrt{1800}$ is ridiculous)

endpoints will be given tomorrow. Test the values tomorrow

finding y using (*)

$y = \frac{600}{\sqrt{1800}} = \frac{\sqrt{1800}}{3} = \frac{\sqrt{9 \times 200}}{3}$

$= \frac{\cancel{3}\sqrt{100 \times 2}}{\cancel{3}}$

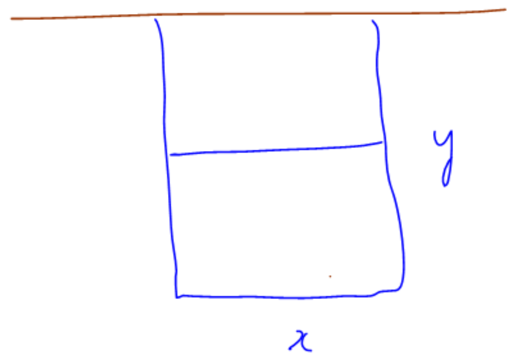
$= 10\sqrt{2}$

$\sqrt{1800} = \sqrt{900 \times 2}$

$= 30\sqrt{2}$

$\therefore$ The dimensions are

$30\sqrt{2}$ m by $10\sqrt{2}$ m.



$F = 2x + 2y$

$F(x) = 2x + \frac{1200}{x}$

$F'(x) = 2 - \frac{1200}{x^2}$

$\Rightarrow 2 - \frac{1200}{x^2} = 0$

$\Rightarrow x = \pm\sqrt{600}$ (neg is rid.)

finding y using (*)

$y = \frac{600}{\sqrt{600}} = \sqrt{600}$

$= 10\sqrt{6}$

$\therefore$ The dimensions are

$\sqrt{600}$ m by $\sqrt{600}$ m

($10\sqrt{6}$ by $10\sqrt{6}$)

16. A new product has just come onto the market and is a big hit. The success, however, does not last too long. Within a year, sales/day have dropped drastically. We are given that the number of sales/**day**, measured in tens of thousands, of the product is represented by the function $n(t) = -50(e^{-4t} - e^{-3t})$ after t **years**. How many days after the product first enters the market is the maximum number of sales/day? (Recall that 365 days = 1 yr.) (Be sure to **prove** your answer provides a maximum)
How many products were sold on that day? **A/6**

(No interval given. The C.V. is where a max is)

usually you will have to deal w/ endpoints

you can use the 2ND derivative test

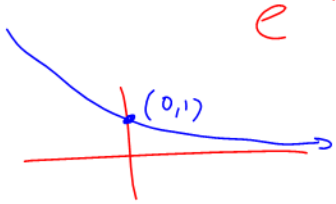
$$n'(t) = -50\left(-4e^{-4t} + 3e^{-3t}\right) \quad \text{set to zero}$$

$$\Rightarrow -50\left(-4e^{-4t} + 3e^{-3t}\right) = 0 \qquad \div -50$$

$$\Rightarrow -4e^{-4t} + 3e^{-3t} = 0$$

$$\Rightarrow e^{-4t}\left(-4 + 3e^{t}\right) = 0$$

$$e^{-4t} \neq 0 \Rightarrow -4 + 3e^{t} = 0$$



$$\Rightarrow 3e^{t} = 4$$

$$e^{t} = \frac{4}{3}$$

$$t = \ln\left(\frac{4}{3}\right)$$

$$= 0.288 \text{ years}$$

$$(= 105 \text{ days})$$

After 105 days the max sales is reached and is

$$n(0.288) = -50\left(e^{-4(0.288)} - e^{-3(0.288)}\right)$$

$$= 5.27 \text{ sales/day}$$

17. a. Determine the derivative of $f(x) = (x+1)^2(3x-5)^4$. Write your answer in simplified factored form.
b. Determine the value(s) of x for which the graph of $f(x)$ has a horizontal tangent.

a) $f'(x) = 2(x+1)(3x-5)^4 + (x+1)^2\left(4(3x-5)^3(3)\right)$

$= 2(x+1)(3x-5)^3\left((3x-5) + 6(x+1)\right)$

$= 2(x+1)(3x-5)^3(9x+1)$

b) set to zero $2(x+1)(3x-5)^3(9x+1) = 0$

$\Rightarrow x = -1, \frac{5}{3}, -\frac{1}{9}.$

D

Happy Studying