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MATH@TD

Name: ________________________

MCV4U - Examiniation (Calculus Half) - Practice

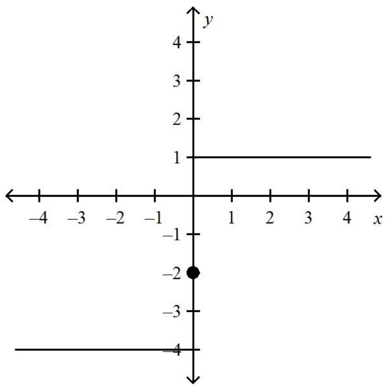
Note: Your actual exam will be similar in length and question type.

PART A - Multiple Choice (10 Marks #1-6 K/1 each, #7-10 T/1 each)
Identify the choice that best completes the statement or answers the question. Place the letter of your choice on the appropriate place on the answer sheet.

1. Determine which expression is the correct rationalization of the numerator of $\dfrac{\sqrt{x}+3}{x-6}$.

   a. $\dfrac{x+3}{\sqrt{x}-6}$
   b. $\dfrac{x+9}{(x-6)(\sqrt{x}+3)}$
   c. $\dfrac{x-9}{(x+6)(\sqrt{x}-3)}$
   d. $\dfrac{x-9}{(x-6)(\sqrt{x}-3)}$

2. Does the following graph represent a function that satisfies the following conditions: $f(0)=-2,\ \lim\limits_{x\to 0} f(x)=-4$?



   a. Yes
   b. No
   c. Only if $f(1)=1$.
   d. There is not enough given information.

3. Determine $\lim\limits_{x\to -3} \dfrac{2x^3-18x}{x+3}$.

   a. 6
   b. –6
   c. 36
   d. –36

4. Let $f(x)=\dfrac{3x^2+1}{4x^2-16}$. What is the equation of the horizontal asymptote of $f(x)$?

   a. $x=\dfrac{3}{4}$
   b. $x=2$
   c. $y=\dfrac{3}{4}$
   d. $y=2$

5. Let $f(x)=\dfrac{x^2-4x-12}{x^2-8x+12}$. Determine the equation(s) of the vertical asymptote(s) of $f(x)$.

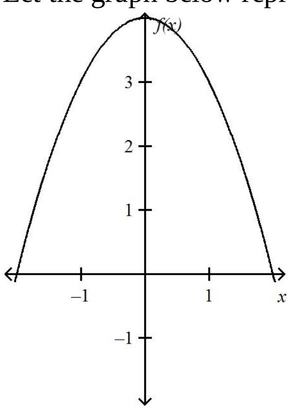
a. $x=2$

b. $x=6$

c. $x=-2$

d. $x=2$ and $x=6$

6. Let the graph below represent $f(x)$. For what values of x is the second derivative positive?



a. f'' is always positive

b. f'' is never positive

c. $-2 < x < 2$

d. $-\infty < x < 0$

7. Which function is continuous?

a. $f(x)=\begin{cases}\dfrac{2}{x-1} & \text{if } x\le 0\\ \dfrac{3x+4}{x+2} & \text{if } x>0\end{cases}$

b. $f(x)=\begin{cases}\dfrac{1}{x-3} & \text{if } x\le 1\\ \dfrac{-1}{x+1} & \text{if } x>1\end{cases}$

c. $f(x)=\begin{cases}\dfrac{x+3}{x} & \text{if } x\le -1\\ \dfrac{1}{x+5} & \text{if } x>-1\end{cases}$

d. $f(x)=\begin{cases}\dfrac{x+2}{x} & \text{if } x<0\\ \dfrac{1}{x+1} & \text{if } x\ge 0\end{cases}$

8. Let the position of a certain particle be described by the function $s(t)=kt^2-(5k+1)t+k$. For which constant value of k is the particle stationary when the time t = 2 s?

a. $k=-\dfrac{1}{5}$

b. $k=1$

c. $k=2$

d. $k=-1$

9. Determine the possible point of inflection of $f(x) = x^3 + 2x^2 - 4x + 7$.

a. $\left(-\dfrac{2}{3},\dfrac{277}{27}\right)$

b. $\left(\dfrac{2}{3},-\dfrac{277}{27}\right)$

c. $\left(\dfrac{2}{3},\dfrac{277}{27}\right)$

d. $\left(-\dfrac{2}{3},-\dfrac{277}{27}\right)$

10. Determine the equation of the tangent to the curve $y = 2^{\sin x}$ at the point with x-coordinate $\frac{\pi}{2}$.

a. $y - 2 = 0$

b. $-x + y - (2 + \frac{\pi}{2}) = 0$

c. $x + y - (2 - \frac{\pi}{2}) = 0$

d. $-x + y + (2 - \frac{\pi}{2}) = 0$

Part B
Full Soutions (40 Marks -- K/20, T/9, A/11,
Organization and Presentation of Work (10 Marks -- C/10)
Write complete and well-written solutions to the following problems in the appropriate spaces on the answer sheets.

11. Using the fact that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, determine $f'(x)$ for the function $f(x) = \tan(x)$. **T/4**

12. Find the derivative of $f(x) = \sqrt{3x - 2}$ from first principles. **K/6**

13. Determine the first derivative of each of the following (simplify your answers as much as possible for **T** points):

a) $h(x) = \left(\dfrac{x-2}{1-x}\right)^2$ **K/2 T/1**

b) $f(x) = 3^{\sqrt{2x-3}}$ **K/2**

c) $g(x) = \cos^2(x) \cdot \ln\left(\sin(x)\right)$ **K/3 T/1**

d) $y = 2x^3 \cdot e^{\sin(x^2)}$ **K/3 T/1**

14. Consider the function $f(x) = x - e^{3x}$.
a. Determine the absolute extrema of the function on the interval $-4 \le x \le 0$. **K/4**
b. Do the absolute extrema change on the interval $-4 \le x \le -1$? Explain. If there is a new absolute extrema, determine it. **T/2**

15. A farmer needs to fence off an area of 600 m^2, making two pens of equal size. He will use an existing wood fence on his farm for one side of the enclosed region and will construct the other pieces of the fence with wire fencing. Find the dimensions of the pens that minimize the length of the new wire fencing. (Be sure to **prove** your answer provides a minimum) **A/5**

16. A new product has just come onto the market and is a big hit. The success, however, does not last too long. Within a year, sales/day have dropped drastically. We are given that the number of sales/**day**, measured in tens of thousands, of the product is represented by the function $n(t) = -50(e^{-4t} - e^{-3t})$ after t **years**. How many days after the product first enters the market is the maximum number of sales/day? (Recall that 365 days = 1 yr.) (Be sure to **prove** your answer provides a maximum)
How many products were sold on that day? **A/6**

17. a. Determine the derivative of $f(x) = (x+1)^2(3x-5)^4$. Write your answer in simplified factored form.
b. Determine the value(s) of x for which the graph of $f(x)$ has a horizontal tangent.