MCV4U - W14: Some Calculus Exam Practice Problems

Full solutions to the following problems will be posted online sometime Tuesday April 15. DON'T JUST WAIT FOR THE SOLUTIONS!!! TRY THEM YOURSELVES....holy cow, try them yourselves.

1. Determine the limit for each of the following:

   a) $\lim\limits_{x\to 3}\sin\left(\frac{\pi}{x}\right)$ b) $\lim\limits_{x\to -2}\frac{3x^2+x-2}{x^2-4}$ c) $\lim\limits_{x\to\infty}\frac{3x-5x^2-2x^4}{3x^4-7}$ d) $\lim\limits_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{x^2-3x+2}$

2. Let $f(x)=-3(2x+4)^{\frac{2}{3}}$. For what values of x is $f(x)$ concave up?

3. Determine the derivative of $f(x)=\sqrt{2x-1}$ from first principles.

4. Differentiate each of the following. State which differentiation rule (or rules) you used in taking the derivative.

   a) $f(x)=3x^3-5x+7x^{-2}$ b) $g(x)=\left(5x^3-3x\right)^3\left(2x^2-x+5\right)^4$

   c) $h(x)=\frac{(3x-1)^2}{2x^2+3}$ d) $f(x)=2^{\sin\left(5x^2\right)}$

   e) $y=\cos(3x)\cdot\sin^3(5x)$ f) $g(x)=\ln\left(\frac{\sin(x)}{3x^3-5x}\right)$

5. Find the points on the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ where the tangent to the curve is parallel to the line $y = 2x$. Show that two of these points have the same tangent.

6. Let $y=\left(\frac{x-1}{x+1}\right)^3$. Determine the equation of the tangent to the curve at $\left(2,\frac{1}{27}\right)$.

7. Find the equation of the tangent line to the curve $f(x) = \dfrac{x^2}{x-6}$ at the point where $x = 3$.

8. Let $f(x) = \dfrac{(1-3x)^2}{(1-4x)^3}$. Determine the intervals of increase or decrease.

9. Investigate whether the graph of the curve $y = x^2 \ln x$ has a point of inflection. If it does, state the coordinates of the point of inflection.

10. 500 g of radioactive material decays exponentially. Amount A of material left after t years is $A(t) = A_0(1.3)^{-t}$. How fast is the material decaying after 5 years?

11. A beam of length $10m$ is supported at one end. If $5kg$ is the uniform weight per metre of length, the bending moment M at a distance x from the end is given by $M = \dfrac{1}{2}lx - \dfrac{1}{2}wx^2$, where l is the beam's length in metres and w is the uniform weight per metre of length. Find the points on the beam at which the bending moment has the maximum value.

12. A closed right circular cylinder is such that the sum of its height and the circumference of its base is 10 m. Find the maximum volume of the cylinder.

13. Cameron needs to fence off an area of 300 m^2 for his two sheep. He will use an existing fence on his farm for one side of the enclosed region and will construct the other three sides with cedar posts. Find the dimensions of the sheep pasture that minimize the length of the new fencing.

14. a. Determine the values of a and b for $f(x) = -2x^3 + ax^2 + bx + 6$ so that $f'(-2) = 0$ and $f'(7) = 0$.
    b. For what values of x is $f(x)$ increasing?

15. Let $f(x) = 2x^3 - 9x^2 - 60x + 1$. Use the second derivative test to determine all local minima of $f(x)$.

16. Use the algorithm for curve sketching to sketch the graph of $f(x) = \dfrac{x-2}{x^2 - 3x - 4}$.