6.8b Linear Combinations and Spanning Sets 2

Yesterday we considered the notion of a **Linear Combination** (***scalar multiplication and vector addition***):

> e.g. Given two vectors $\vec{u}$ and $\vec{v}$, then $\vec{w} = a\vec{u} + b\vec{v}$ is called a linear combination of $\vec{u}$ and $\vec{v}$.

Consider the standard unit vectors for $\mathbb{R}^2$: $\hat{i} = (1,0)$ and $\hat{j} = (0,1)$. We can write any vector in $\mathbb{R}^2$ as a linear combination if the standard unit vectors!!

> Consider $\vec{u} = (x, y)$

Thus the set of vectors $\{\hat{i}, \hat{j}\}$ is called a

Similarly, the set of vectors $\{(1,0,0),(0,1,0),(0,0,1)\}$

Definition 6.8b.1

> Given vectors $\vec{u}$ and $\vec{v}$, we call the set of all possible linear combinations of $\vec{u}$ & $\vec{v}$ the **SPAN** of $\{\vec{u}, \vec{v}\}$.
>
> We write $span\{\vec{u}, \vec{v}\} = \{\vec{w} = a\vec{u} + b\vec{v},\ a, b \in \mathbb{R}\}$
>
> Thus we can write $span\{\hat{i}, \hat{j}\} =$

Now, spanning sets are not

Consider the set $A = \{(1,0),(0,1),(3,2)\}$. Clearly $span\{A\} =$

BUT

Definition 6.8b.2

We call a set which is **NOT linearly dependent** a **linearly independent set**.

Definition 6.8b.3

A **linearly dependent set of vectors**, $A = \{\vec{u}, \vec{v}\}$, which **spans another set** $B = \{\vec{w} = a\vec{u} + b\vec{v},\ a, b, \in \mathbb{R}\}$ is called a **BASIS** of that set.

So, the set $A = \{\hat{i} = (1,0),\ \hat{j} = (0,1)\}$ is a basis for

And now for some more difficult stuff…

Example 6.8b.1

Show $span\left\{\left(1,2\right),\left(-1,3\right)\right\} = \mathbb{R}^2$

In $\mathbb{R}^3$, and two non-collinear vectors form a plane.

Picture

Note: the set $A = \{\vec{u}, \vec{v}\}$ is **NOT**

Three **non-coplanar** vectors **WILL** span $\mathbb{R}^3$.

Picture:

An Important Question:

How can we determine if three vectors $\left(\text{in } \mathbb{R}^3\right)$ are non-coplanar?

Answer:

Example 6.8b.2

From your text: Pg. 41 #13a

Show that the vectors $(-1,2,3)$, $(4,1,-2)$ and $(-14,1,16)$ do not lie in the same plane.

Class/Homework for Section 6.8 (pt. 2)

Pg. 340 - 342 #1 – 6, 7, 10, 11, 8, 9, 12b, 13b, 14, 15