Shame on Errington for skipping class!

6.2 Vector Addition (and subtraction)

Consider the following:

> Fred walks $2.5km$ E, and then turns 30° toward the North and walks a further $3.2km$. How far is Fred from his starting point, and in what direction?

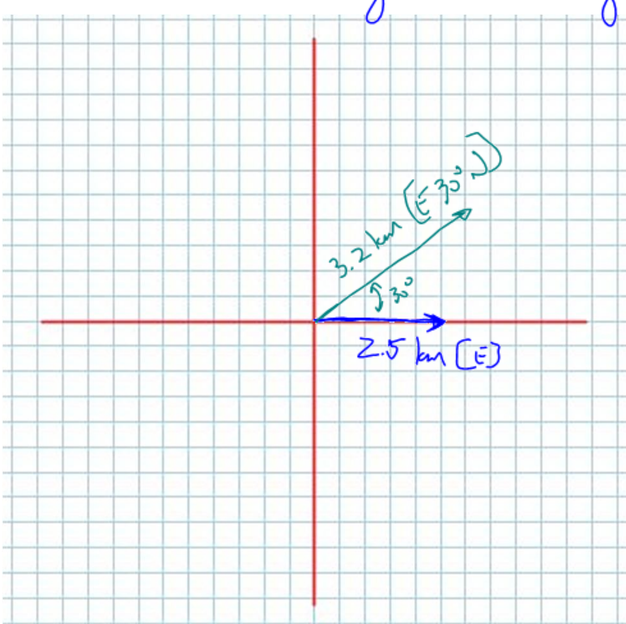
A Vector is the solution to this problem!

There are two approaches to a "geometric representation" of Fred's situation:

↳ pictures are your friend

Position Diagram

↳ vectors "begin" at the origin

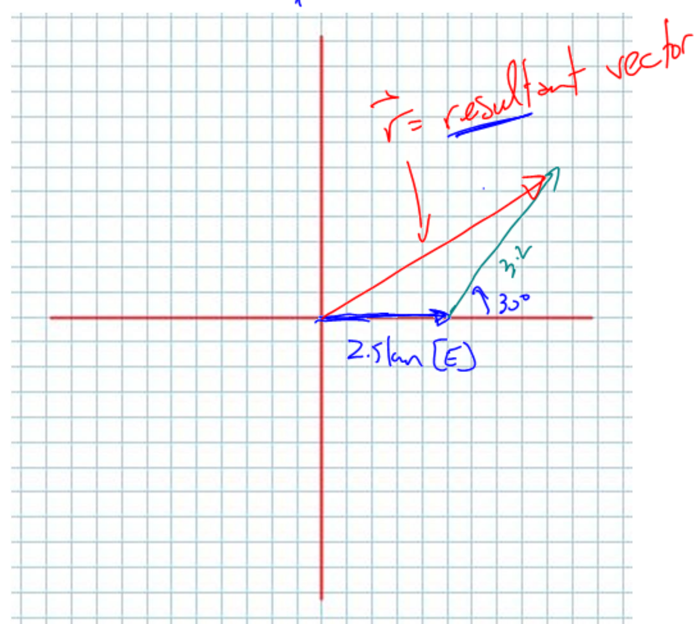


"tail to tail" addition

(nice for "free-body" diagram)

Triangle Diagram

(we "map out" the situation)



"head to tail" addition

nice because resultant vector is easy to see.

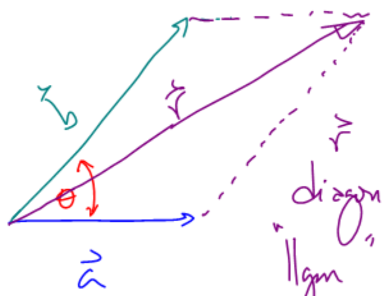
we calculate

- $|\vec{r}|$ using cosine law
- direction of $\vec{r}$ using sine law

Vector Addition

Parallelogram Law

(tail to tail)

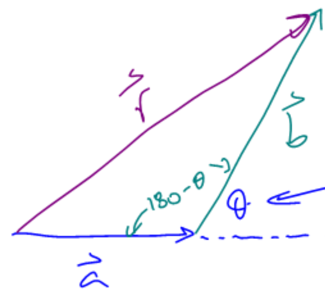


$\vec{r}$ is the diagonal of the "llgm"

θ is the angle between the vectors $\vec{a}$ & $\vec{b}$ (really - it's the vectors' directions!)

Triangle Law

(head to tail)

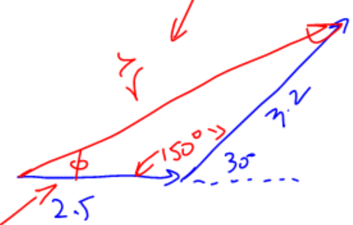


angle between the directions of $\vec{a}$ & $\vec{b}$

Solving Fred's Problem

Draw a picture

use cosine law to get $|\vec{r}|$



once we have $|\vec{r}|$, use sine law to get ϕ

Solving for the resultant vector $\vec{r}$, we need $\boxed{2}$ things.

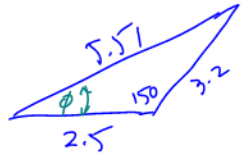
① Magnitude ② Direction.

$$|\vec{r}|^2 = (2.5)^2 + (3.2)^2 - 2(2.5)(3.2)\cos(150)$$

$$\Rightarrow \quad |\vec{r}| = \sqrt{(\qquad \text{"} \qquad)}$$

$$= 5.51 \text{ km}.$$

Getting ϕ



$$\frac{\sin(\phi)}{3.2} = \frac{\sin(150)}{5.51}$$

$$\phi = \sin^{-1}\left(\frac{(3.2)(\sin(150))}{5.51}\right)$$

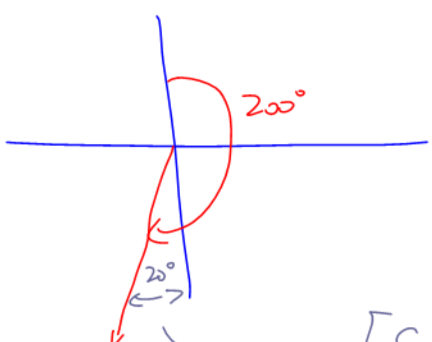
$$= 17^\circ$$

$\therefore$ Fred is 5.51 km [E 17° N] from his starting point

(OR $\vec{r}$ = 5.51 km [E 17° N])

either are acceptable conclusions

Note: We can use **bearings** to describe direction. Bearings measure angles from N (0 degrees) rotating clockwise. For example, a bearing of 200° looks like:



or [S 20° W]

I prefer this notation as it's more descriptive.

Vector Subtraction

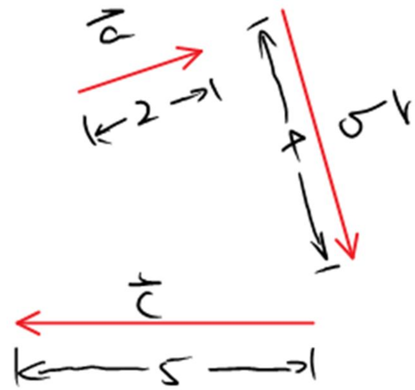
$\vec{b}$ in opposite direction

We can write $\vec{a}-\vec{b}$ as $\vec{a}+\left(-\vec{b}\right)$ and simply use "vector addition".

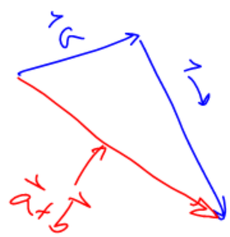
Example 6.2.1

Given vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, draw:

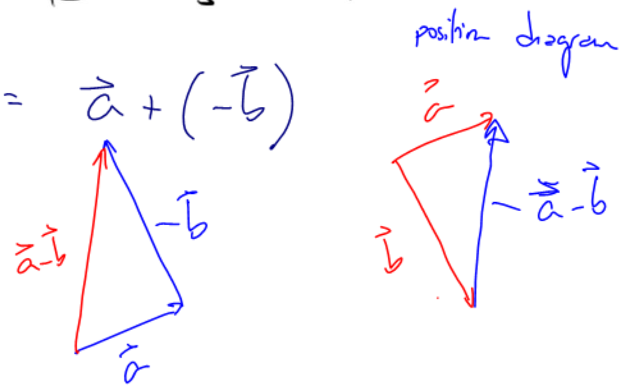
i) $\vec{a}+\vec{b}$ ii) $\vec{a}-\vec{b}$ iii) $\vec{a}+\vec{b}-\vec{c}$



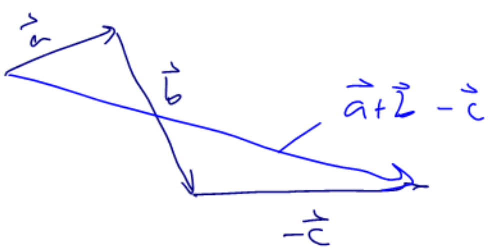
i)



ii) $\vec{a}-\vec{b} = \vec{a}+(-\vec{b})$

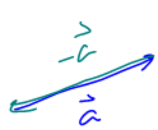


iii)



Note: A **KEY** aspect of vectors is about to be presented to you…pay attention!!

Consider $\vec{a}+\left(-\vec{a}\right)$



$\vec{a}+(-\vec{a}) = \vec{0}$

the zero vector

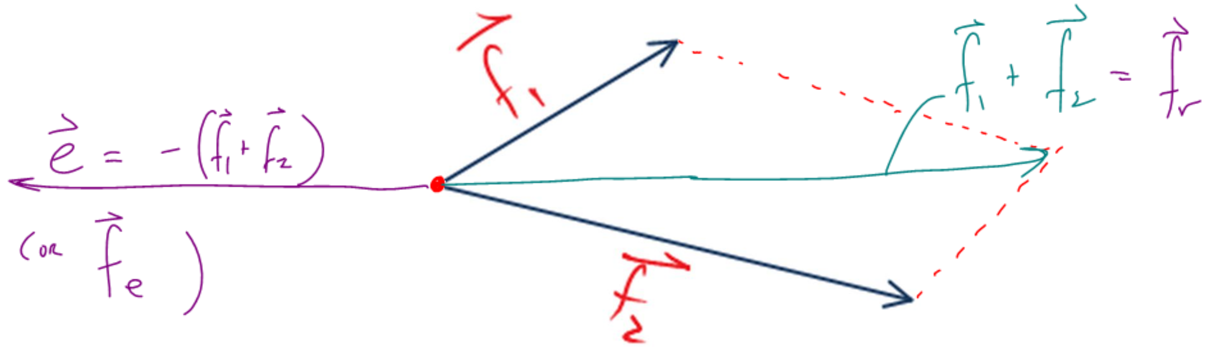
VITAL FOR ALGEBRA.

One final note (related to the zero vector):

The Equilibrant (an idea from physics)

Consider the position diagram of forces acting on a (point) body:
Draw the **resultant** force, and ask yourself the intriguing question:

"What force would be needed to keep the body from moving?"

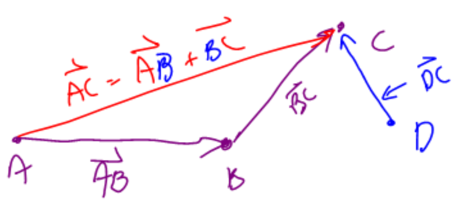


$\vec{e}$ (or $\vec{f}_e$) is the equilibrant force (cancelling the resultant $\vec{f}_r = \vec{f}_1 + \vec{f}_2$)

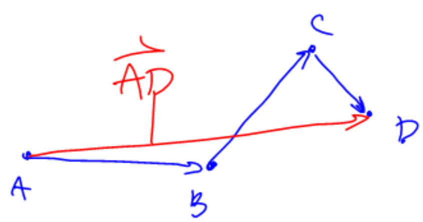
$$\vec{f}_e = -\vec{f}_r$$

Example 6.2.2

Write as a single vector: $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{DC}$ (hint: draw a picture)



Note $-\overrightarrow{DC} = \overrightarrow{CD}$



$$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{DC}$$

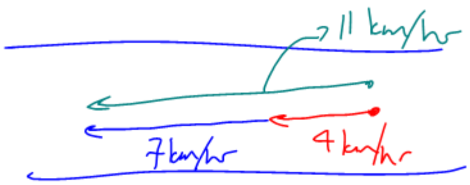
$$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$$

Example 6.2.3

From your text: Pg. 291 #9a

In still water, Maria can paddle at the rate of 7 km/h. The current in which she paddles has a speed of 4 km/h.

a. At what velocity does she travel downstream? — with the current



11 km/hr is her resultant velocity

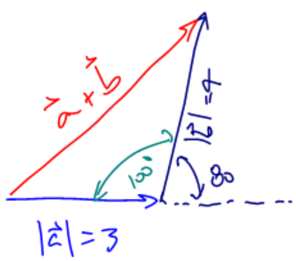
Example 6.2.4

Two vectors $\vec{a}$ $\left(|\vec{a}| = 3cm\right)$ and $\vec{b}$ $\left(|\vec{b}| = 4cm\right)$ have an angle between them of 80°.

Determine $|\vec{a}+\vec{b}|$.

Notes: we don't have specific directions for $\vec{a}$ nor $\vec{b}$

when drawing a picture, assume a direction (for $\vec{a}$)



$$|\vec{a}+\vec{b}| = \sqrt{3^2+4^2-2(3)(4)\cos(100)}$$

$$= 5.4 \text{ cm.}$$

Class/Homework for Section 6.2

Pg. 290 – 292 #1 – 14 (Ex 4 on Pg. 287 is awesome)