6.4 Properties of Vector Addition and Scalar Multiplication *(6.3 is too simple)*

We want to use **vectors** to "**do mathematics**". To be able to perform mathematical operations, we need a ***structure*** (a set of mathematical rules) inside of which we "do the math". Such a mathematical **structure** can be considered **an algebra**.

Today we will see **SEVEN RULES** making up a basic structure (algebra) which will allows us to perform Vector Addition and Scalar Multiplication of vectors.

Given vectors $\vec{a}, \vec{b},$ and $\vec{c}$ and scalars $k, m,$ and n:

1) The Commutative Property of Vector Addition

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

2) The Associative Property of Vector Addition

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{c}) + \vec{b}$$

3) The Distributive Property of Scalar Multiplication Over Addition

$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$$

4) The Additive Inverse Element

there exists   such that

Given $\vec{a}$, $\exists\ -\vec{a}$ s.t.

$$\vec{a} + (-\vec{a}) = -\vec{a} + \vec{a} = \vec{0}$$

5) The Additive Identity Element for Vector Addition

Given $\vec{b}$, $\exists$ $\vec{0}$ st.

$$\vec{b}+\vec{0} = \vec{0}+\vec{b} = \vec{b}$$

6) The Associative Law for Scalar Multiplication

$$m(n\vec{a}) = (mn)\vec{a} = n(m\vec{a})$$

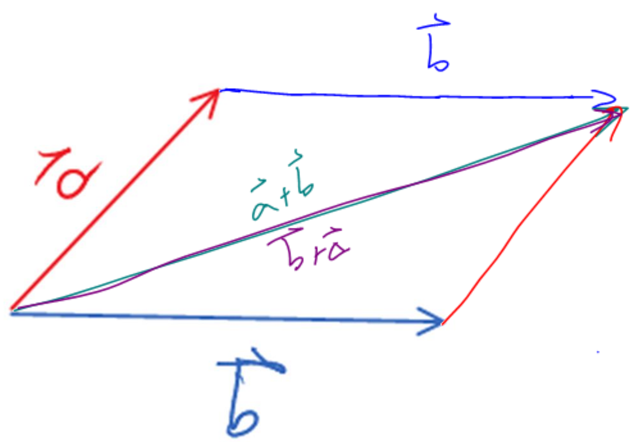
7) The Distributive Law for Scalar Multiplication

$$(k+m)\vec{a} = k\vec{a} + m\vec{a}$$

The **seven rules** above constitute the **ALGEBRAIC STRUCTURE** inside of which we will work with vectors. Note that the structure is very familiar!

A geometric proof of Rule 1

Given $\vec{a}$ and $\vec{b}$ in their standard positions: $\vec{a}+\vec{b} = \vec{b}+\vec{a}$



Example 6.3.1

Given $\vec{x}=5\hat{i}+2\hat{j}-3\hat{k}$, and $\vec{y}=-2\hat{i}+7\hat{j}+5\hat{k}$, determine:

a) $\vec{x}+\vec{y}$

$= 5\hat{i}+2\hat{j}-3\hat{k}+(-2\hat{i}+7\hat{j}+5\hat{k})$

Commutative prop

$= 5\hat{i}-2\hat{i}+2\hat{j}+7\hat{j}-3\hat{k}+5\hat{k}$

rule 7 $= 3\hat{i}+9\hat{j}+2\hat{k}$

b) $2\vec{x}-4\vec{y}$

$= 2(5\hat{i}+2\hat{j}-3\hat{k})-4(-2\hat{i}+7\hat{j}+5\hat{k})$

distributive

$= 10\hat{i}+4\hat{j}-6\hat{k}+8\hat{i}-28\hat{j}-20\hat{k}$

$= 18\hat{i}-24\hat{j}-26\hat{k}$

Example 6.3.2

From your text: Pg. 307 #5

Show $\overrightarrow{PQ}=\left(\overrightarrow{RQ}+\overrightarrow{SR}\right)+\overrightarrow{TS}+\overrightarrow{PT}$

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

RHS $= (\overrightarrow{RQ}+\overrightarrow{SR})+\overrightarrow{TS}+\overrightarrow{PT}$

$= \overrightarrow{PT}+\overrightarrow{TS}+\overrightarrow{SR}+\overrightarrow{RQ}$

$= \overrightarrow{PQ}$

Example 6.3.3

Given $\vec{a} = 4\vec{x} - \vec{y}$, and $\vec{b} = 2\vec{x} + 3\vec{y}$, write $\vec{x}$ and $\vec{y}$ **in terms of** $\vec{a}$ and $\vec{b}$.

for $\vec{x}$ eliminate $\vec{y}$

Consider

$$3\vec{a} + \vec{b} = 12\vec{x} - 3\vec{y} + 2\vec{x} + 3\vec{y}$$

$$\Rightarrow 14\vec{x} = 3\vec{a} + \vec{b}$$

$$\Rightarrow \vec{x} = \frac{3}{14}\vec{a} + \frac{1}{14}\vec{b}$$

for $\vec{y}$ eliminate $\vec{x}$

Consider

$$\vec{a} - 2\vec{b} = 4\vec{x} - \vec{y} - 4\vec{x} - 6\vec{y}$$

$$\Rightarrow -7\vec{y} = \vec{a} - 2\vec{b}$$

$$\Rightarrow \vec{y} = -\frac{1}{7}\vec{a} + \frac{2}{7}\vec{b}$$

Class/Homework for Section 6.3

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