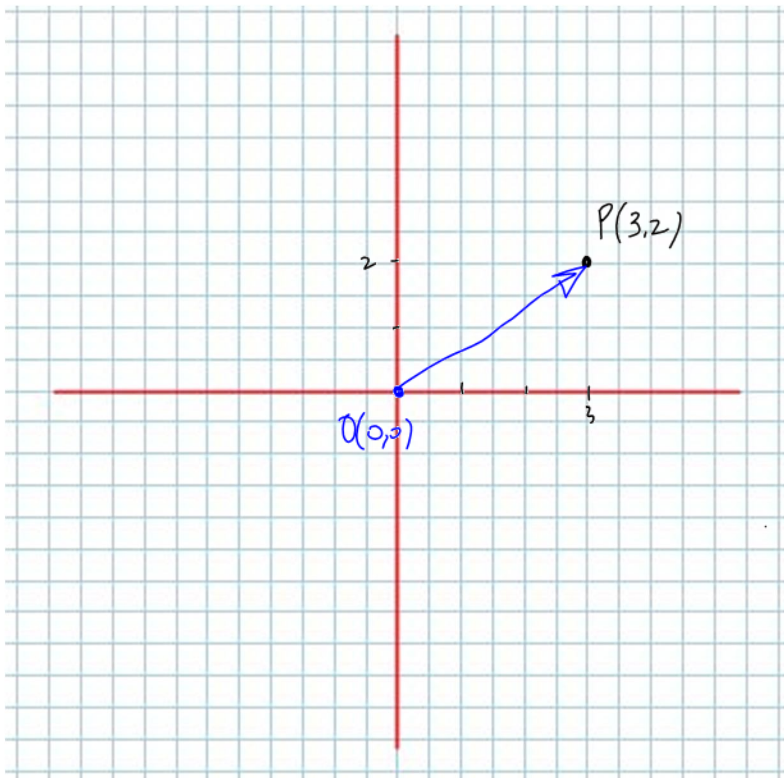
6.5 Vectors in 2D and 3D

In this sections we will (hopefully) begin to see **DEEP** connections between algebra and geometry.

Consider the *x*-*y* plane (also known as the Cartesian plane), with the point $P(3,2)$



Note: If we draw a vector from $O(0,0)$ "pointing to *P*" we create the vector $\overrightarrow{OP}$ which is said to be in STANDARD POSITION (tail is at the origin O(0,0))

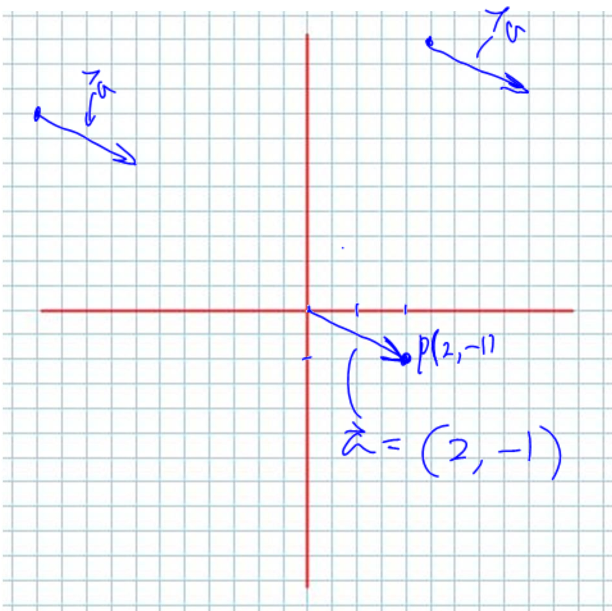
We write

$\overrightarrow{OP} = (3,2)$

Note: Be sure to recognize the **context** that you are working in!!

Example 6.5.1

Draw the vector $\vec{a} = (2, -1)$ in standard position. (tail at origin



Note: We call the **coordinates of the point** (*the axis numbers*) the **components** of the vector.

For $\vec{a} = (2, -1)$

x-component 2

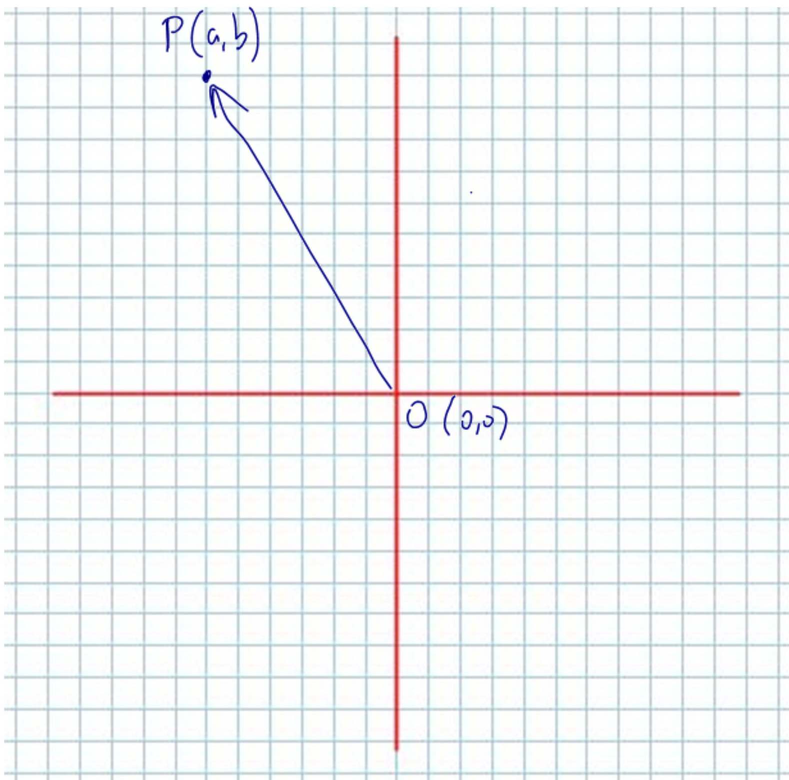
y-component −1

Another Note

Vectors in standard position are UNIQUE. There is only one vector in standard position which 'points' from the origin to the point (2, −1)

The General Position Vector

Consider the position vector $\overrightarrow{OP} = (a,b)$



Note that $\overrightarrow{OP} = (a,b)$ points

Uniquely

at the point $P(a,b)$

We call the collection of **ALL POINTS** in the *x*-*y* plane $\mathbb{R}^2$ ("are two")

We write $\mathbb{R}^2 = \{ (x,y) \mid x \in \mathbb{R} , y \in \mathbb{R} \}$

Key Note $\mathbb{R}^2$ is a set of POINTS NOT VECTORS

BUT every point in $\mathbb{R}^2$ can be uniquely associated with a vector in standard position

Moving up to $\mathbb{R}^3$

$\mathbb{R}^2$ is called a two-dimensional **space** because it can be (***fully!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!***) defined using two coordinates. If we add a third coordinate we can discuss (mathematically) a three-dimensional **space** which we call $\mathbb{R}^3$, and which we denote:

$$\mathbb{R}^3 = \left\{ (x, y, z) \middle| x \in \mathbb{R},\ y \in \mathbb{R},\ z \in \mathbb{R} \right\}$$

In $\mathbb{R}^3$ we write the origin as $O(0,0,0)$ and a general point as $P(a, b, c)$

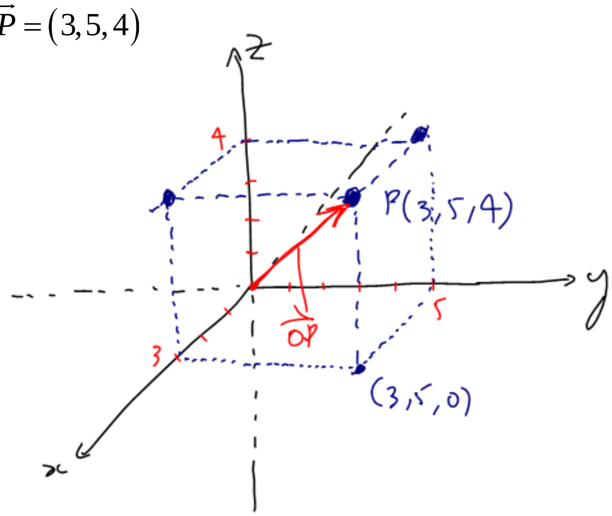
Thus we have the UNIQUE general position vector

$$\vec{OP} = (a, b, c)$$

Representing Vectors in $\mathbb{R}^3$

Example 6.5.2

Draw the vector $\overrightarrow{OP} = (3,5,4)$



Note: The axes form 3 planes

$$\left.\begin{array}{l} xy \text{ plane} \Rightarrow z = 0 \\ xz \text{ plane} \Rightarrow y = 0 \\ yz \text{ plane} \Rightarrow x = 0 \end{array}\right\} \text{eq'ns of "axis-planes"}$$

Example 6.5.3

Determine the equation of the plane containing the points $E(0,0,3)$, $F(2,0,3)$, $G(2,5,3)$, and $H(0,5,3)$.

$z = 3$

Determine the equation of the plane containing points *O*, *E*, *F*

$O(0,0,0)$

$y = 0$

Which points are in the *y-z* plane?

$x = 0$

$E(0,0,3) \qquad H(0,5,3)$

Example 6.5.4

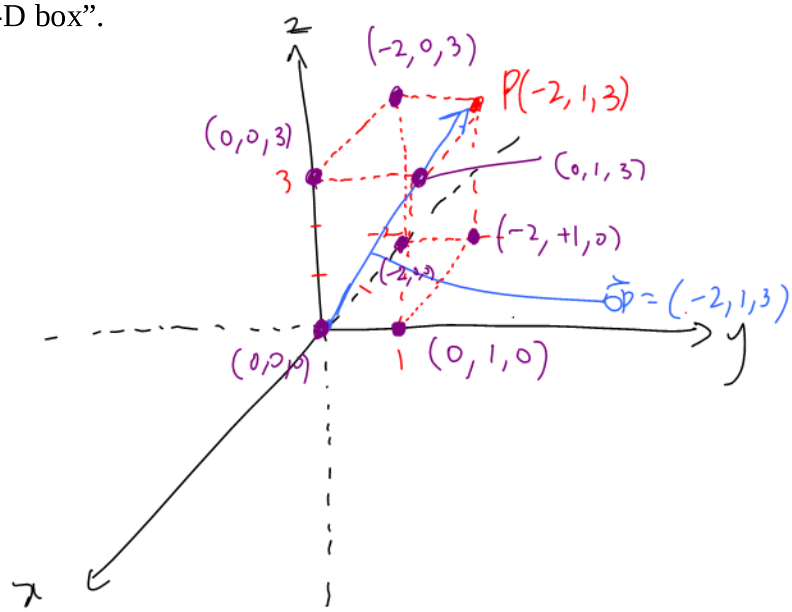
Given $\overrightarrow{OP} = (2, b, c)$ and $\overrightarrow{OP} = (a, 3, 0)$ determine *a*, *b*, and *c*. **Why can the three unknowns be determined?**

Vectors in standard position are UNIQUE

$a = 2$
$b = 3$
$c = 0$

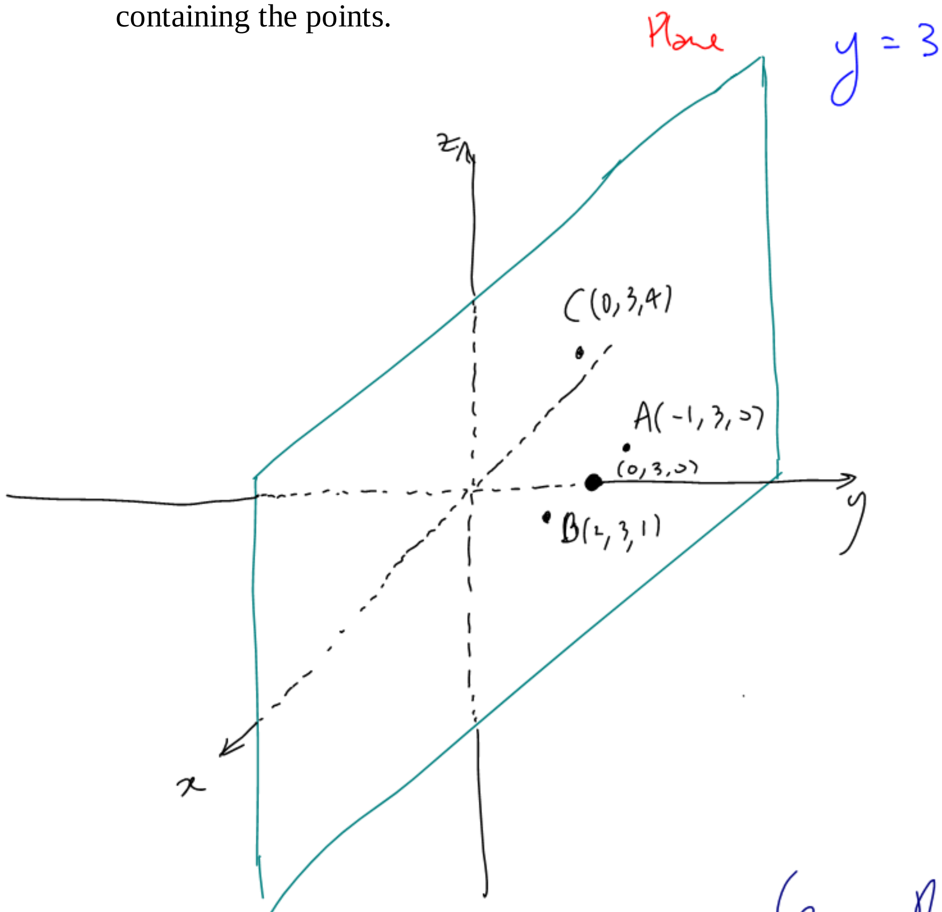
Example 6.5.5

Plot the point $P(-2,1,3)$ and draw the associated position vector. Label each "corner" of your "3-D box".



Example 6.5.6

Plot the points $A(-1,3,0)$, $B(2,3,1)$ and $C(0,3,4)$ and give an equation for the plane containing the points.



Note:
The plane $y=3$ does NOT contain the vectors $\vec{OA}$, $\vec{OB}$ nor $\vec{OC}$

(even though it contains the points A, B, C)

Class/Homework for Section 6.5

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