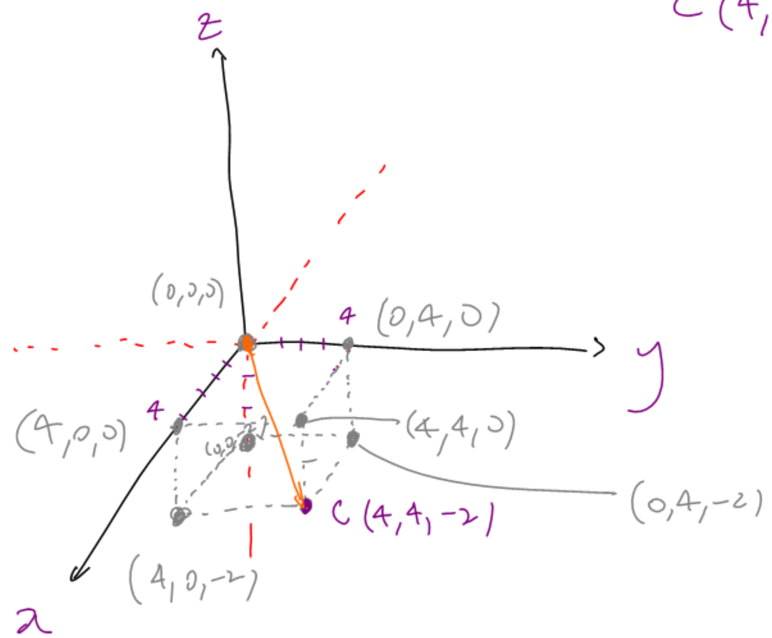
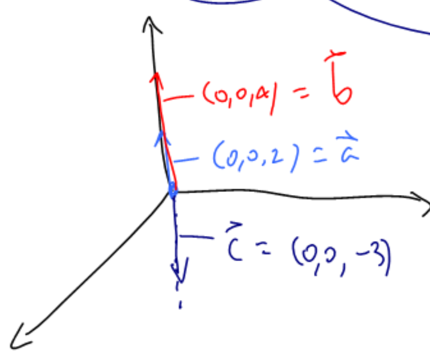
5. Locate the points $A(4, -4, -2)$, $B(-4, 4, 2)$, and $C(4, 4, -2)$ using coordinate axes that you construct yourself. Draw the corresponding rectangular box (prism) for each, and label the coordinates of its vertices.

$C\,(4, 4, -2)$



7. a. Name three vectors with their tails at the origin and their heads on the z-axis.
   b. Are the vectors you named in part a, collinear? Explain.



same "direction" (line)

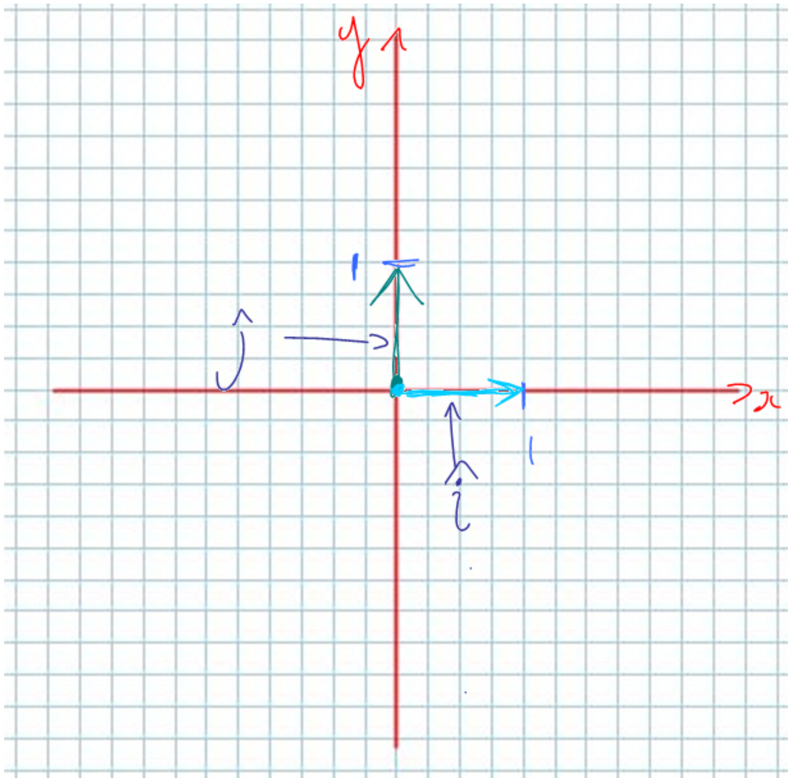
yes

$\vec{b} = 2\vec{a}$

$\vec{c} = -\frac{3}{2}\vec{c}$

6.6 Algebraic Operations with Vectors in $\mathbb{R}^2$

We will begin by considering two **Very Special** vectors.



UNIT VECTORS

have a magnitude of 1

$|\hat{\imath}| = 1$

$|\hat{\jmath}| = 1$

$\hat{\imath} = (1, 0)$

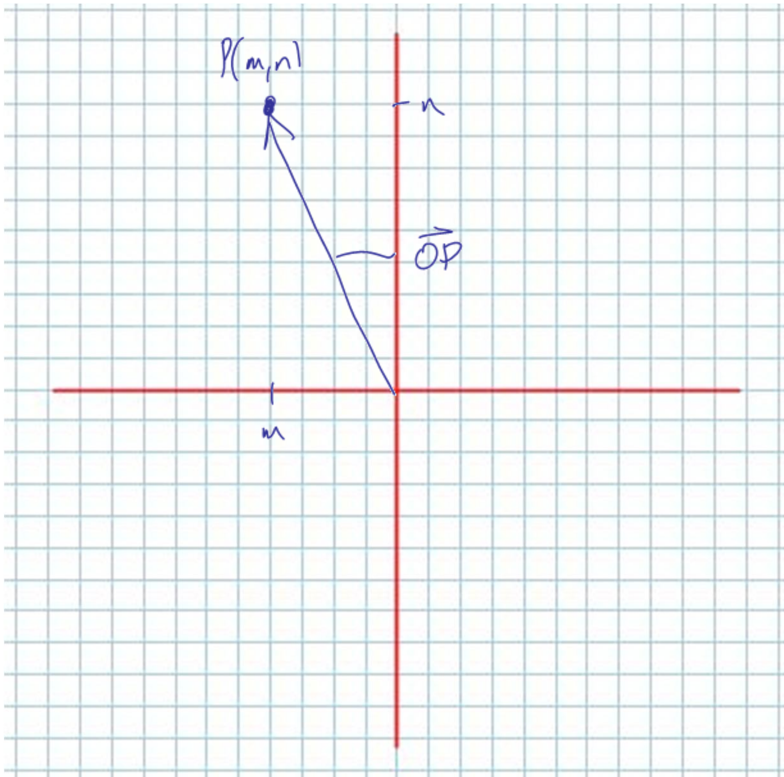
$\hat{\jmath} = (0, 1)$

The Standard Unit Vectors are **beautiful** because they are so easy to “scale”. For example, consider the vector $\vec{a} = (7,0)$. We can write $\vec{a} = 7\hat{\imath}$

Another example would be rewriting $\vec{b} = (0.-3)$ as $\vec{b} = -3\hat{\jmath}$

Consider the general position vector for $\mathbb{R}^2$ $\overrightarrow{OP} = (m, n)$, $(\text{where } m, n \in \mathbb{R})$.

A picture:



$$\overrightarrow{OP} = (m, n) = (m, 0) + (0, n) \quad \left(= m(1,0) + n(0,1) \right)$$
$$= m\hat{i} + n\hat{j}$$

Huge Insight

All of $\mathbb{R}^2$ can be obtained, constructed using the two standard unit vectors

We say **ANY** vector in $\mathbb{R}^2$ can be **uniquely** written as a **Linear Comination** of the standard unit vectors.

Definition 6.6.1 Given vectors $\vec{a}$ & $\vec{b}$ and scalars m, n we call the vector $\vec{v} = m\vec{a} + n\vec{b}$ is a linear combination of $\vec{a}$ & $\vec{b}$. (Linear combos are constructed through scalar multiplication and addition)

Adding Vectors Algebraically

Example 6.6.1

Given $\overrightarrow{OA} = (3,1)$, and $\overrightarrow{OB} = (2,2)$, determine $\overrightarrow{OA} + \overrightarrow{OB}$

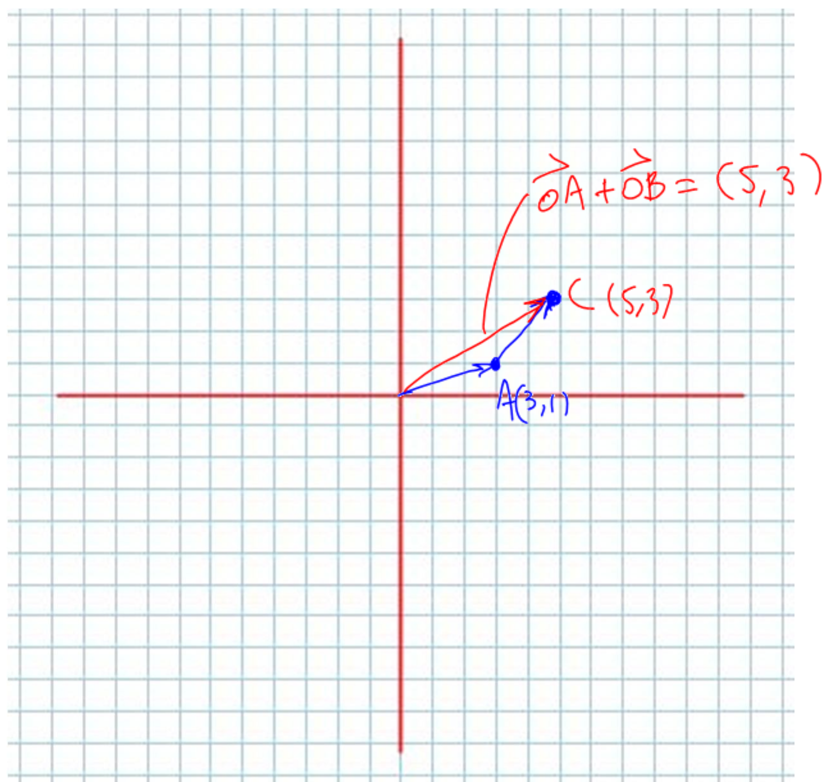
Picture

Algebraically

$\overrightarrow{OA} + \overrightarrow{OB}$

$= (3,1) + (2,2)$

$= (5,3)$



Note:

We add vectors **component-wise**

(x component + x component, y comp + y comp)

Example 6.6.2

Given $\vec{a} = (3, -5)$, and $\vec{b} = (-8, -2)$, determine:

i) $\vec{a} + \vec{b}$ ii) $\vec{b} - \vec{a}$

i) $= (3-8, -5-2)$

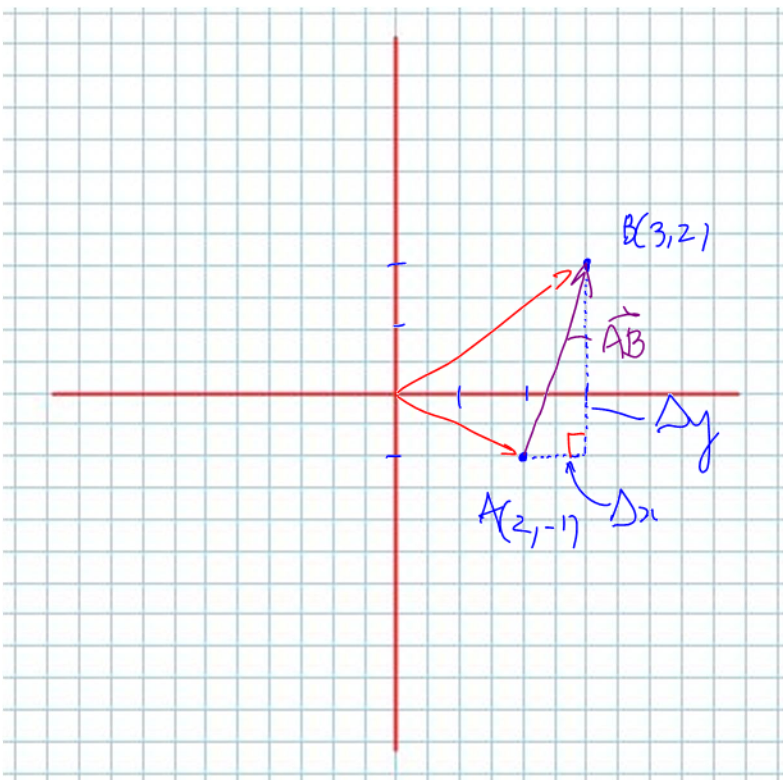
$= (-5, -7)$

ii) $= (-8-3, -2+5)$

$= (-11, +3)$

Example 6.6.3 (*this is an important one…well they all are, but this one especially)*

Given the **points** $A(2,-1)$ and $B(3,2)$, draw vector $\overrightarrow{AB}$ and determine its **components**.



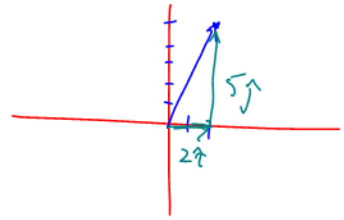
$\vec{AB} = (1, 3)$

$= (3-2, 2-(-1))$

change in x   change in y

(head minus tail)

Magnitude of a vector algebraically



$|5\vec{j}| =$

$5|\vec{j}| = 5$

Consider the position vector $\overrightarrow{OA} = (2,5)$.

$$|\overrightarrow{OA}| = \sqrt{2^2 + 5^2}$$
$$= \sqrt{29} \doteq 5.4$$

In general, for some vector $\overrightarrow{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$

$$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Consider now a position vector $\vec{a} = (x, y)$

$$|\vec{a}| = \sqrt{x^2 + y^2}$$

Example 6.6.4

Given $\vec{a} = (3,-1)$ and $\vec{b} = (-2,4)$ find $|\vec{a} - 2\vec{b}|$.

$$\vec{a} - 2\vec{b} = (3,-1) - 2(-2,4)$$
$$= (3,-1) - (-4,8)$$
$$= (7,-9)$$

$$\therefore |\vec{a} - 2\vec{b}| = \sqrt{7^2 + (-9)^2} = \sqrt{130}$$

Class/Homework for Section 6.6

Pg. 324 – 326 #1 – 17