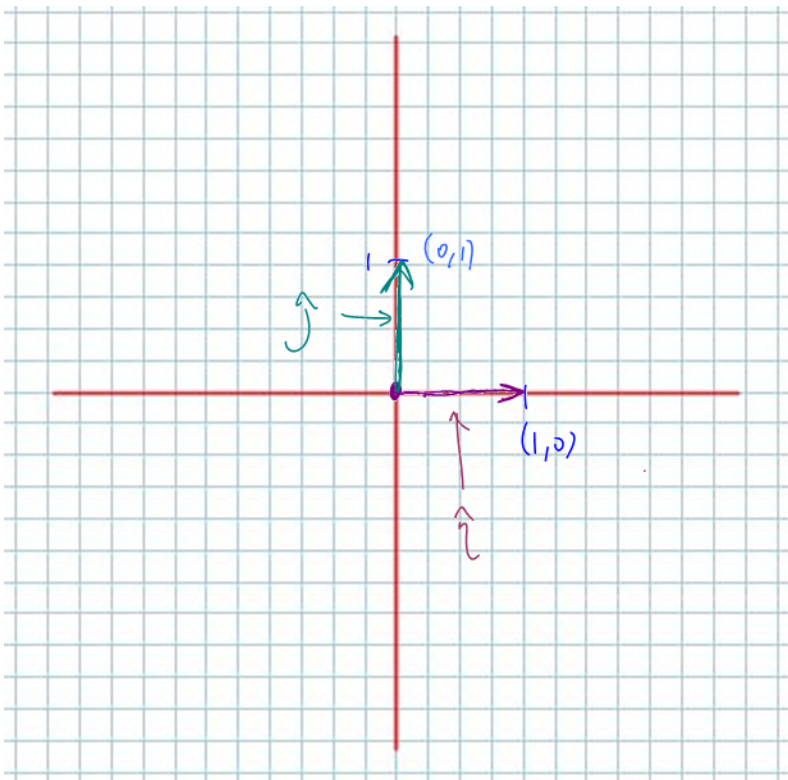
6.6 Algebraic Operations with Vectors in $\mathbb{R}^2$

We will begin by considering two **Very Special** vectors.



Vectors with a magnitude of 1 unit are called unit vectors

Notation: $\hat{a}$ ← unit vector

$\hat{i} = (1,0)$

$\hat{j} = (0,1)$

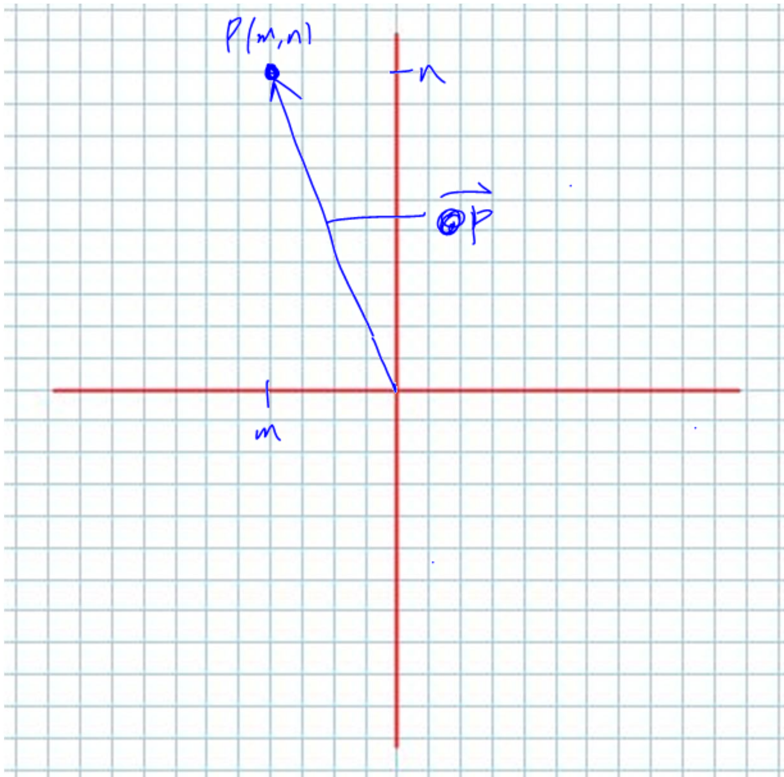
the standard unit vectors for $\mathbb{R}^2$

The Standard Unit Vectors are **beautiful** because they are so easy to "scale". For example, consider the vector $\vec{a} = (7,0)$. We can write $\vec{a} = 7(1,0) = 7\hat{i}$

Another example would be rewriting $\vec{b} = (0.-3)$ as $\vec{b} = -3(0,1) = -3\hat{j}$

Consider the general position vector for $\mathbb{R}^2$ $\overrightarrow{OP} = (m, n)$, $(\text{where } m, n \in \mathbb{R})$.

A picture:



$\overrightarrow{OP} = (m, n)$

$= (m, 0) + (0, n)$

$= m\hat{i} + n\hat{j}$

Huge Insight

All of $\mathbb{R}^2$ can be obtained, or constructed using the standard unit vectors $\hat{i} = (1, 0)$ and $\hat{j} = (0, 1)$

Combination

We say **ANY** vector in $\mathbb{R}^2$ can be **uniquely** written as a **Linear** ~~**Comination**~~ of the standard unit vectors.

Definition 6.6.1 Given vector $\vec{a}$ & $\vec{b}$ and scalars m, n then the vector $\vec{v} = m\vec{a} + n\vec{b}$ is called a linear combination of $\vec{a}$ & $\vec{b}$ (A linear combination uses scalar multiplication and addition)

Adding Vectors Algebraically

Example 6.6.1

Given $\overrightarrow{OA} = (3,1)$, and $\overrightarrow{OB} = (2,2)$, determine $\overrightarrow{OA} + \overrightarrow{OB}$

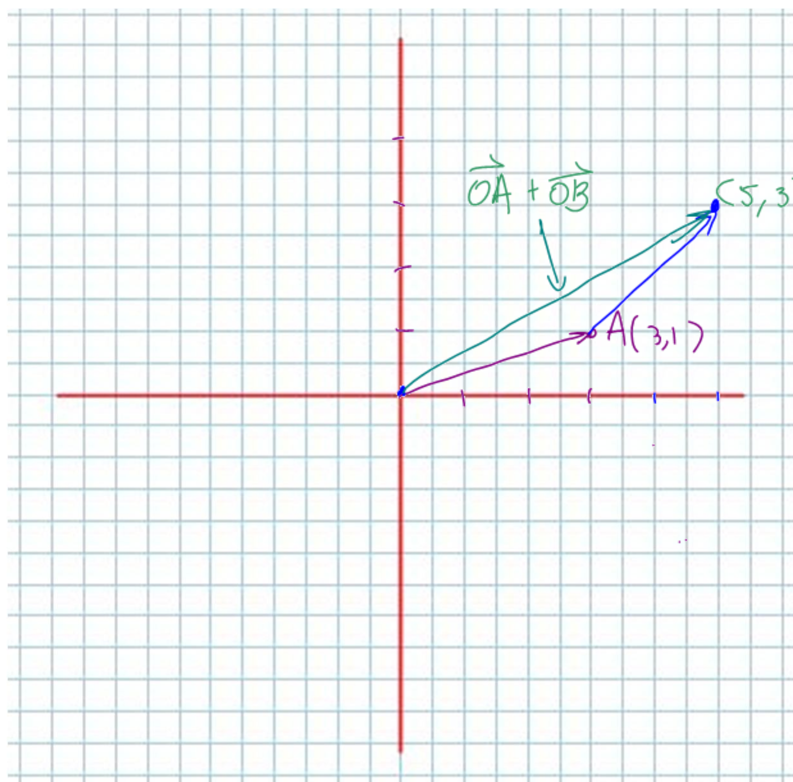
Picture

$\overrightarrow{OA} + \overrightarrow{OB}$

$= (3,1) + (2,2)$

$= (5,3)$

(Notice: $(3+2, 1+2) = (5,3)$!)



Note: $(5,3) = (3+2, 1+2)$

We add vectors **component-wise**

Example 6.6.2

Given $\vec{a} = (3,-5)$, and $\vec{b} = (-8,-2)$, determine:

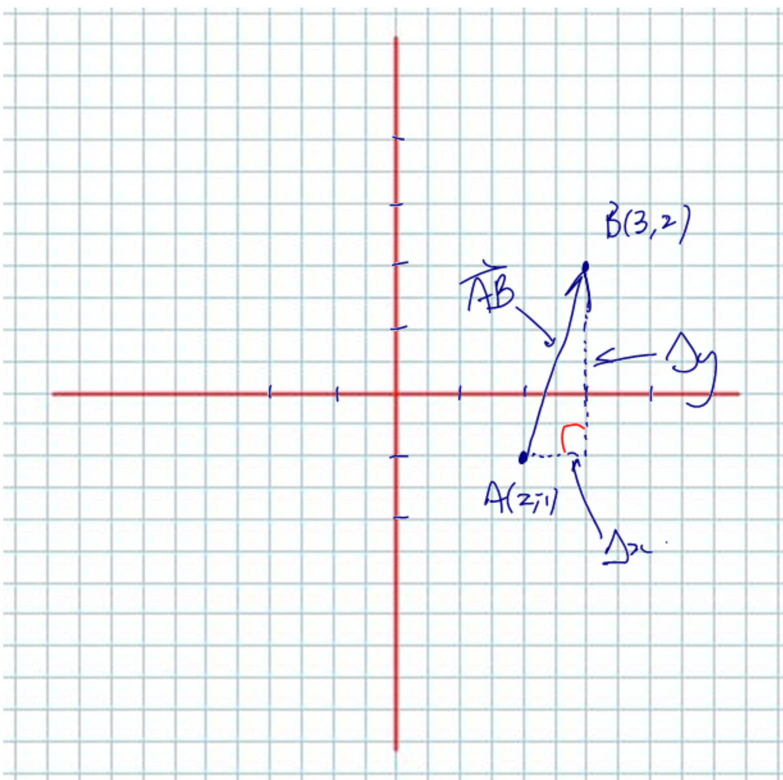
i) $\vec{a}+\vec{b}$ ii) $\vec{b}-\vec{a}$

$$\vec{a}+\vec{b} = (3,-5)+(-8,-2)$$
$$= (3+(-8), -5+(-2))$$
$$= (-5,-7) \quad (\text{or } -5\hat{i}-7\hat{j})$$

$$\vec{b}-\vec{a} = (-8,-2)-(3,-5)$$
$$= (-8-3, -2+5)$$
$$= (-11, 3) \quad (\text{or } -11\hat{i}+3\hat{j})$$

Example 6.6.3 (*this is an important one…well they all are, but this one especially*)

Given the **points** $A(2,-1)$ and $B(3,2)$, draw vector $\overrightarrow{AB}$ and determine its **components**.



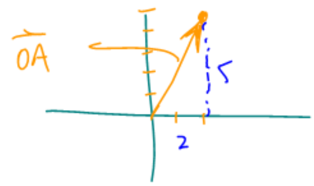
To get from A to B we move a bit in the x direction and a bit in the y direction

$$\overrightarrow{AB} = (3-2, 2-(-1))$$
$$= (1, 3)$$

"Δx" "Δy"

("head" - "tail")

Note: $\overrightarrow{AB}$ is the hypotenuse of a right angle Δ



Magnitude of a vector algebraically

Consider the position vector $\overrightarrow{OA}=(2,5)$.

$$|\overrightarrow{OA}| = \sqrt{2^2+5^2}$$
$$= \sqrt{29}$$

(by Pythagorean Theorem)

In general, for some vector $\overrightarrow{AB}$ with endpoints $A(x_1,y_1)$ and $B(x_2,y_2)$

$$|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

(just $\sqrt{\Delta x^2+\Delta y^2}$)

Consider now a position vector $\vec{a}=(x,y)$

$$|\vec{a}| = \sqrt{x^2+y^2}$$

Example 6.6.4

Given $\vec{a}=(3,-1)$ and $\vec{b}=(-2,4)$ find $\left|\vec{a}-2\vec{b}\right|$.

$$\vec{a}-2\vec{b} = (3,-1)-2(-2,4)$$
$$= (3,-1)-(-4,8)$$
$$= (3+4,\ -1-8)$$
$$= (7,-9)$$

$$\therefore\ |\vec{a}-2\vec{b}|$$
$$= |(7,-9)|$$
$$= \sqrt{7^2+(-9)^2}$$
$$= \sqrt{130}$$

Class/Homework for Section 6.6

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