7. Given $\vec{x} = 2\vec{i} - \vec{j}$ and $\vec{y} = -\vec{i} + 5\vec{j}$, find a vector equivalent to each of the following:
   a. $3\vec{x} - \vec{y}$
   b. $-(\vec{x} + 2\vec{y}) + 3(-\vec{x} - 3\vec{y})$
   c. $2(\vec{x} + 3\vec{y}) - 3(\vec{y} + 5\vec{x})$

8. Using $\vec{x}$ and $\vec{y}$ given in question 7, determine each of the following:
   a. $|\vec{x} + \vec{y}|$  b. $|\vec{x} - \vec{y}|$  c. $|2\vec{x} - 3\vec{y}|$  d. $|3\vec{y} - 2\vec{x}|$

8b) $|\vec{x} - \vec{y}| = |(2\hat{i} - \hat{j}) - (-\hat{i} + 5\hat{j})|$

$= |3\hat{i} - 6\hat{j}| = |(3, -6)|$

$= \sqrt{3^2 + (-6)^2}$

$= \sqrt{45} = 3\sqrt{5}$

13. Determine the value of $x$ and $y$ in each of the following:
   a. $3(x, 1) - 5(2, 3y) = (11, 33)$
   b. $-2(x, x + y) - 3(6, y) = (6, 4)$

13 a) $3(x, 1) - 5(2, 3y) = (11, 33)$

$\Rightarrow (3x, 3) - (10, 15y) = (11, 33)$

$\Rightarrow (3x - 10, 3 - 15y) = (11, 33)$

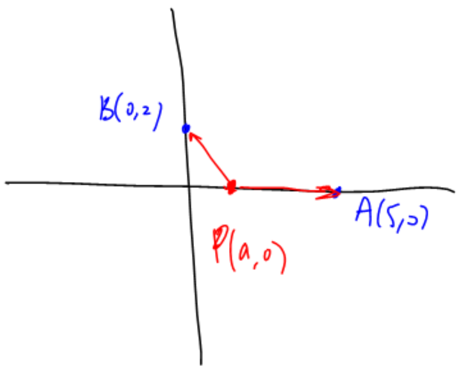
$\Rightarrow 3x - 10 = 11$  $3 - 15y = 33$

$\Rightarrow x = 7$  $y = -2$

15. $A(5, 0)$ and $B(0, 2)$ are points on the $x$- and $y$-axes, respectively.

a. Find the coordinates of point $P(a, 0)$ on the $x$-axis such that $|\overrightarrow{PA}| = |\overrightarrow{PB}|$.

b. Find the coordinates of a point on the $y$-axis such that $|\overrightarrow{QB}| = |\overrightarrow{QA}|$.




$\overrightarrow{PA} = (5-a, 0)$

$\overrightarrow{PB} = (-a, 2)$

$A(x_1, y_1)$

$B(x_2, y_2)$

$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$

We want $|\overrightarrow{PA}| = |\overrightarrow{PB}|$

$\Rightarrow |(5-a, 0)| = |(-a, 2)|$

$\sqrt{(5-a)^2 + 0^2} = \sqrt{(-a)^2 + (2)^2}$

$\Rightarrow (5-a)^2 = a^2 + 4$

$\Rightarrow a^2 - 10a + 25 = a^2 + 4$

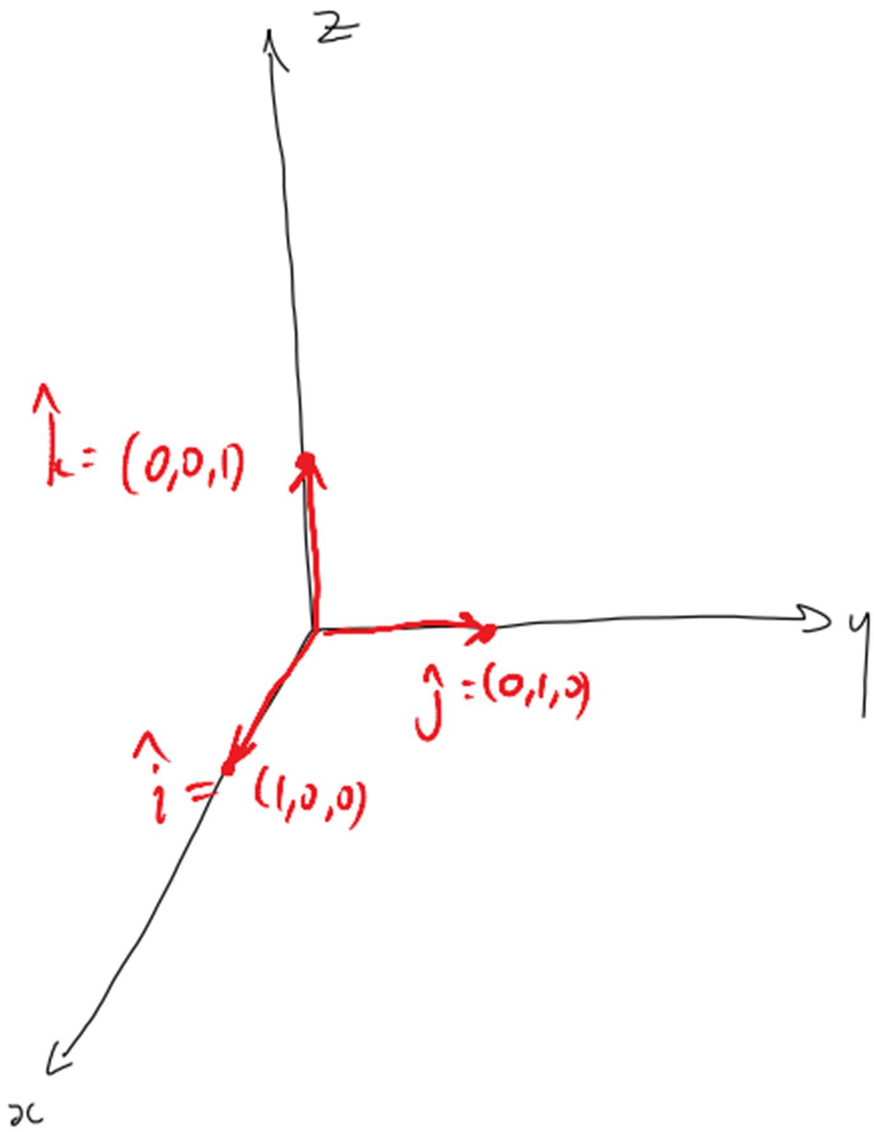
$\Rightarrow -10a = -21$

$\Rightarrow a = 2.1$ $\quad \therefore P(2.1, 0)$ is our point.

# 6.7 Algebraic Operations with Vectors in $\mathbb{R}^3$

Today's lesson is an extension (into the third dimension…***ominous music plays***) of what we saw in section 6.6.

Consider the sketch:




We call the vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ the STANDARD UNIT VECTORS for $\mathbb{R}^3$

Recall

$$\mathbb{R}^3 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$$

As in $\mathbb{R}^2$ we have a **unique** association between points and position vectors in $\mathbb{R}^3$. That is, given a point $P(a,b,c)$ we can uniquely define the position vector $\overrightarrow{OP}=(a,b,c)$. Furthermore, we can write $\overrightarrow{OP}$ as a **linear combination** of the standard unit vectors in $\mathbb{R}^3$:

$$\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$$

Consider the ***general*** vector $\overrightarrow{AB}$ in $\mathbb{R}^3$ where the points $A(x_1,y_1,z_1)$, and $B(x_2,y_2,z_2)$ are the tail and tip of $\overrightarrow{AB}$ respectively. We can write

$$\overrightarrow{AB} = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$$

Further, by Pythagorus,

$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Finally, for a general position vector $\vec{v}=(a,b,c)$

$$\left|\vec{v}\right| = \sqrt{a^2+b^2+c^2}$$

Patrick - 5000

**Class/Homework for Section 6.7**

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