14. Given the points $A(-2, 1, 3)$ and $B(4, -1, 3)$, determine the coordinates of the point on the x-axis that is equidistant from these two points.

same distance

Let $P(a, 0, 0)$ be the point

$$|\vec{PA}| = |\vec{PB}|$$

$$|(-2-a, 1, 3)| = |(4-a, -1, 3)|$$

$$\Rightarrow \sqrt{(-2-a)^2 + (1)^2 + (3)^2} = \sqrt{(4-a)^2 + (-1)^2 + (3)^2}$$

$$\Rightarrow (-2-a)^2 + 1 + 9 = (4-a)^2 + 1 + 9$$

$$4 + 4a + a^2 = 16 - 8a + a^2$$

$$12a = 12$$

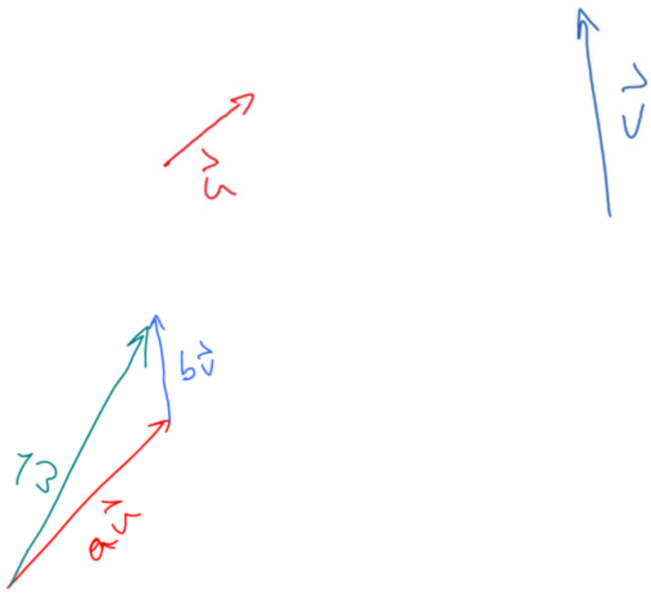
$$a = 1$$

$\therefore$ $P(1, 0, 0)$ is equidistant from A & B

6.8 Linear Combinations and Spanning Sets

Given the non collinear vectors $\vec{u}$ and $\vec{v}$, and the scalars *a* and *b*, we can construct a third vector $\vec{w} = a\vec{u} + b\vec{v}$ (we call $\vec{w}$ a **linear combination** of vectors $\vec{u}$ and $\vec{v}$) .

Picture:



Since $\vec{w}$ is a **linear combination** of $\vec{u}$ and $\vec{v}$ we say that the set of vectors $\{\vec{w},\vec{u},\vec{v}\}$ form a **linear dependent set**. Now, because $\vec{u}$ and $\vec{v}$ are **not collinear**, we call the set $\{\vec{u},\vec{v}\}$ a **linearly independent set.**

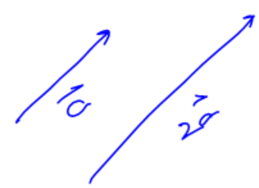
Note: Collinear simply means lying along the same direction

Further; If we can write

$\vec{u} = a\vec{v}$, then

(a is a scalar)

$\{\vec{u},\vec{v}\}$ is linearly dependent



$\vec{a}$ & $2\vec{a}$ are "collinear"

Example 6.8.1

Show that $\vec{w}=(2,-1)$ can be written as a linear combination of $\vec{u}=(3,3)$ and $\vec{v}=(1,2)$.

Note: If $\vec{w}$ is a linear combo of $\vec{u}$ & $\vec{v}$, $\exists$ (there exists) scalars a, b ($\in \mathbb{R}$) s.t. (such that) $\vec{w} = a\vec{u} + b\vec{v}$

Let $\vec{w} = a\vec{u} + b\vec{v}$, a, b are scalars

$\Rightarrow (2,-1) = a(3,3) + b(1,2)$

$\Rightarrow (2,-1) = (3a, 3a) + (b, 2b)$

$\Rightarrow (2,-1) = (3a+b, 3a+2b)$

$\Rightarrow 3a + b = 2$ ①

& $3a + 2b = -1$ ②

② - ① $\Rightarrow b = -3$ sub $b=-3$ into ①

$\Rightarrow a = \frac{5}{3}$

$\therefore \vec{w} = \frac{5}{3}\vec{u} - 3\vec{v}$

Example 6.8.2

Show that $\vec{w} = (2,-1)$ cannot be written as a linear combination of $\vec{x} = (1,3)$ and $\vec{y} = (-2,-6)$.

Assume we can! (proof by Contradiction)

Assume $\exists$ scalars a & $b \in \mathbb{R}$ st.

$$\vec{w} = a\vec{x} + b\vec{y}$$

$$\Rightarrow (2,-1) = a(1,3) + b(-2,-6)$$

$$\Rightarrow (2,-1) = (a-2b,\ 3a-6b)$$

$$\Rightarrow \begin{aligned} a - 2b &= 2 \quad (1) \\ 3a - 6b &= -1 \quad (2) \end{aligned}$$

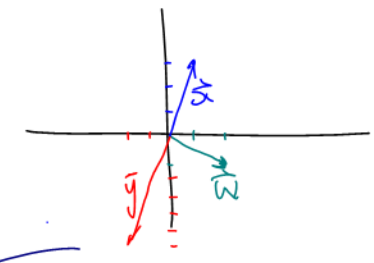
$(1)\times 3 - (2)$ $\quad 0 = 7$ inconsistent

$\Rightarrow$ Contradiction

$\Rightarrow \nexists$ scalars $a, b \in \mathbb{R}$ st

$$\vec{w} = a\vec{x} + b\vec{y} \quad \square$$

Picture



Since $\vec{x} = -\frac{1}{2}\vec{y}$

— $\{\vec{x}, \vec{y}\}$ is linearly dependent

and $\vec{w} \neq a\vec{x}$

$\Rightarrow \nexists$ scalars $a, b \in \mathbb{R}$ st.

$\vec{w} = a\vec{x} + b\vec{y}$

Class/Homework for Section 6.8 (pt. 1)

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