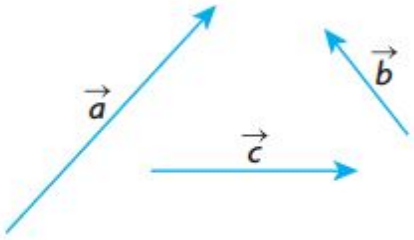
6.4 Properties of Vector Addition and Scalar Multiplication

Problems taken from the Nelson text: Pg. 307

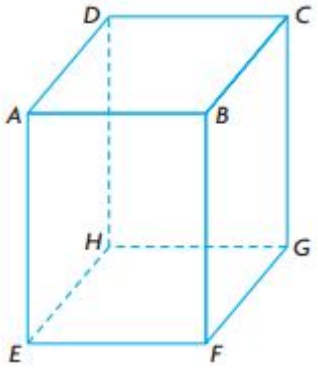
1. If * is an operation on a set, S, the element x, such that $a * x = a$, is called the identity element for the operation *.
   a. For the addition of numbers, what is the identity element?
   b. For the multiplication of numbers, what is the identity element?
   c. For the addition of vectors, what is the identity element?
   d. For scalar multiplication, what is the identity element?

3. Redraw the following three vectors and illustrate the associative law.



4. With the use of a diagram, show that the distributive law, $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$, holds where $k < 0, k \in \mathbf{R}$.

6. *ABCDEFGH* is a rectangular prism.
   a. Write a single vector that is equivalent to $\overrightarrow{EG} + \overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{DC}$.
   b. Write a vector that is equivalent to $\overrightarrow{EG} + \overrightarrow{GD} + \overrightarrow{DE}$.
   c. Is it true that $|\overrightarrow{HB}| = |\overrightarrow{GA}|$? Explain.



7. Write the following vector in simplified form:

$3(\vec{a} - 2\vec{b} - 5\vec{c}) - 3(2\vec{a} - 4\vec{b} + 2\vec{c}) - (\vec{a} - 3\vec{b} + 3\vec{c})$

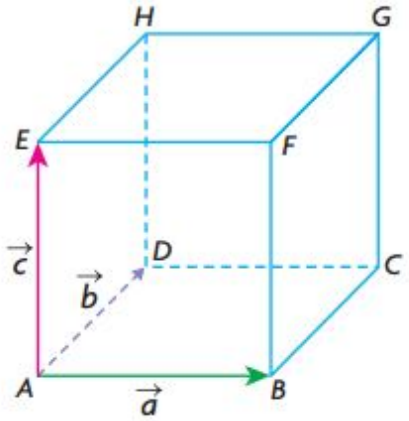
8. If $\vec{a} = 3\vec{i} - 4\vec{j} + \vec{k}$ and $\vec{b} = -2\vec{i} + 3\vec{j} - \vec{k}$, express each of the following in terms of $\vec{i}, \vec{j}$, and $\vec{k}$.

   a. $2\vec{a} - 3\vec{b}$ b. $\vec{a} + 5\vec{b}$ c. $2(\vec{a} - 3\vec{b}) - 3(-2\vec{a} - 7\vec{b})$

9. If $2\vec{x} + 3\vec{y} = \vec{a}$ and $-\vec{x} + 5\vec{y} = 6\vec{b}$, express $\vec{x}$ and $\vec{y}$ in terms of $\vec{a}$ and $\vec{b}$.

10. If $\vec{x} = \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z}$, $\vec{x} - \vec{y} = \vec{a}$, and $\vec{y} - \vec{z} = \vec{b}$, show that $\vec{a} = -\frac{1}{3}\vec{b}$.

11. A cube is constructed from the three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, as shown below.



   a. Express each of the diagonals $\overrightarrow{AG}$, $\overrightarrow{BH}$, $\overrightarrow{CE}$, and $\overrightarrow{DF}$ in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.

   b. Is $|\overrightarrow{AG}| = |\overrightarrow{BH}|$? Explain.

Answers to Selected Problems

6. a. $\overrightarrow{EC}$
b. $\vec{0}$
c. Yes, the diagonals of a rectangular prism are of equal length.

7. $-4\vec{a} + 9\vec{b} - 24\vec{c}$

8. a. $12\vec{i} - 17\vec{j} + 5\vec{k}$
b. $-7\vec{i} + 11\vec{j} - 4\vec{k}$
c. $= -6\vec{i} + 13\vec{j} - 7\vec{k}$

9. $\vec{x} = \frac{5}{13}\vec{a} - \frac{18}{13}\vec{b}$,
$\vec{y} = \frac{1}{13}\vec{a} + \frac{12}{13}\vec{b}$

10. $\vec{a} = \vec{x} - \vec{y}$
$= \frac{2}{3}\vec{y} + \frac{1}{3}\vec{z} - (\vec{b} + \vec{z})$
$= \frac{2}{3}\vec{y} - \frac{2}{3}\vec{z} - \vec{b}$
$= \frac{2}{3}(\vec{y} - \vec{z}) - \vec{b}$
$= \frac{2}{3}\vec{b} - \vec{b}$
$= -\frac{1}{3}\vec{b}$

11. a. $\overrightarrow{AG} = \vec{a} + \vec{b} + \vec{c}$,
$\overrightarrow{BH} = -\vec{a} + \vec{b} + \vec{c}$,
$\overrightarrow{CE} = -\vec{a} - \vec{b} + \vec{c}$,
$\overrightarrow{DF} = \vec{a} - \vec{b} + \vec{c}$

b. $|\overrightarrow{AG}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$
$= |-\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$
$= |\overrightarrow{BH}|^2$
$\therefore |\overrightarrow{AG}| = |\overrightarrow{BH}|$