6.5 Vectors in 2D and 3D

Problems taken from the Nelson text: Pg. 316 – 318

2. a. Describe in your own words what it means for a point and its associated vector to be uniquely represented in R^3.
   b. Suppose that $\overrightarrow{OP} = (a, -3, c)$ and $\overrightarrow{OP} = (-4, b, -8)$. What are the corresponding values for a, b, and c? Why are we able to be certain that the determined values are correct?

3. a. The points $A(5, b, c)$ and $B(a, -3, 8)$ are located at the same point in R^3. What are the values of a, b, and c?
   b. Write the vector corresponding to $\overrightarrow{OA}$.

5. Locate the points $A(4, -4, -2)$, $B(-4, 4, 2)$, and $C(4, 4, -2)$ using coordinate axes that you construct yourself. Draw the corresponding rectangular box (prism) for each, and label the coordinates of its vertices.

6. a. On what axis is $A(0, -1, 0)$ located? Name three other points on this axis.
   b. Name the vector $\overrightarrow{OA}$ associated with point A.

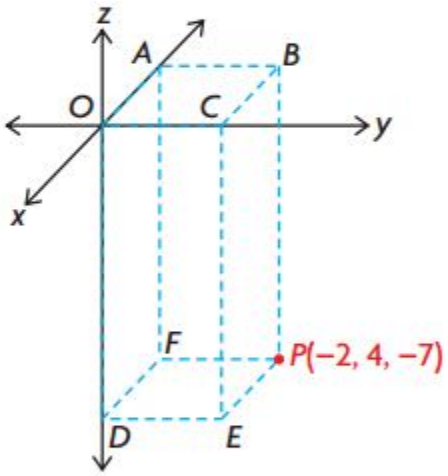
7. a. Name three vectors with their tails at the origin and their heads on the z-axis.
   b. Are the vectors you named in part a. collinear? Explain.
   c. How would you represent a general vector with its head on the z-axis and its tail at the origin?

9. a. Draw a set of x-, y-, and z-axes and plot the following points: $A(3, 2, -4)$, $B(1, 1, -4)$, and $C(0, 1, -4)$.
   b. Determine the equation of the plane containing the points A, B, and C.

14. a. What is the equation of the plane that contains the points $M(1, 0, 3)$, $N(4, 0, 6)$, and $P(7, 0, 9)$? Explain your answer.

    b. Explain why the plane that contains the points M, N, and P also contains the vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$, and $\overrightarrow{OP}$.

15. The point $P(-2, 4, -7)$ is located in R^3 as shown on the coordinate axes below.



    a. Determine the coordinates of points A, B, C, D, E, and F.

    b. What are the vectors associated with each of the points in part a.?

    c. How far below the xy-plane is the rectangle $DEPF$?

    d. What is the equation of the plane containing the points B, C, E, and P?

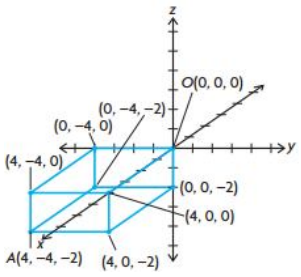
    e. Describe mathematically the set of points contained in rectangle $BCEP$.

Answers to Selected Problems

3. **a.** $a = 5$, $b = -3$, and $c = 8$.
b. $(5, -3, 8)$

15. **a.** $A(-2, 0, 0)$, $B(-2, 4, 0)$, $C(0, 4, 0)$, $D(0, 0, -7)$, $E(0, 4, -7)$, $F(-2, 0, -7)$
b. $\overrightarrow{OA} = (-2, 0, 0)$, $\overrightarrow{OB} = (-2, 4, 0)$, $\overrightarrow{OC} = (0, 4, 0)$, $\overrightarrow{OD} = (0, 0, -7)$, $\overrightarrow{OE} = (0, 4, -7)$, $\overrightarrow{OF} = (-2, 0, -7)$
c. 7 units
d. $y = 4$
e. Every point contained in rectangle $BCEP$ has y-coordinate equal to 4, and so is of the form $(x, 4, z)$, where x and z are real numbers such that $-2 \le x \le 0$ and $-7 \le z \le 0$.

5.



6. **a.** $A(0, -1, 0)$ is located on the y-axis. $B(0, -2, 0)$ $C(0, 2, 0)$, and $D(0, 10, 0)$ are three other points on this axis.
b. $\overrightarrow{OA} = (0, -1, 0)$, the vector with tail at the origin $O(0, 0, 0)$ and head at A.

7. **a.** Answers may vary. For example: $\overrightarrow{OA} = (0, 0, 1)$, $\overrightarrow{OB} = (0, 0, -1)$, $\overrightarrow{OC} = (0, 0, -5)$
b. Yes, these vectors are collinear (parallel), as they all lie on the same line, in this case the z-axis.
c. A general vector lying on the z-axis would be of the form $\overrightarrow{OA} = (0, 0, a)$ for any real number a. Therefore, this vector would be represented by placing the tail at O and the head at the point $(0, 0, a)$ on the z-axis.