# 6.7 Algebraic Operations with Vectors in $\mathbb{R}^3$

Problems taken from the Nelson Text: Pg. 332 – 333

1. a. Write the vector $\overrightarrow{OA} = (-1, 2, 4)$ using the standard unit vectors.
   b. Determine $\left|\overrightarrow{OA}\right|$.

2. Write the vector $\overrightarrow{OB} = 3\vec{i} + 4\vec{j} - 4\vec{k}$ in component form and calculate its magnitude.

3. If $\vec{a} = (1, 3, -3)$, $\vec{b} = (-3, 6, 12)$, and $\vec{c} = (0, 8, 1)$, determine $\left|\vec{a} + \frac{1}{3}\vec{b} - \vec{c}\right|$.

5. Given $\vec{x} = (1, 4, -1)$, $\vec{y} = (1, 3, -2)$, and $\vec{z} = (-2, 1, 0)$, determine a vector equivalent to each of the following:
   a. $\vec{x} - 2\vec{y} - \vec{z}$
   b. $-2\vec{x} - 3\vec{y} + \vec{z}$
   c. $\frac{1}{2}\vec{x} - \vec{y} + 3\vec{z}$
   d. $3\vec{x} + 5\vec{y} + 3\vec{z}$

6. Given $\vec{p} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{q} = -\vec{i} - \vec{j} + \vec{k}$, determine the following in terms of the standard unit vectors.
   a. $\vec{p} + \vec{q}$ b. $\vec{p} - \vec{q}$ c. $2\vec{p} - 5\vec{q}$ d. $-2\vec{p} + 5\vec{q}$

7. If $\vec{m} = 2\vec{i} - \vec{k}$ and $\vec{n} = -2\vec{i} + \vec{j} + 2\vec{k}$, calculate each of the following:
   a. $|\vec{m} - \vec{n}|$ b. $|\vec{m} + \vec{n}|$ c. $|2\vec{m} + 3\vec{n}|$ d. $|-5\vec{m}|$

8. Given $\vec{x} + \vec{y} = -\vec{i} + 2\vec{j} + 5\vec{k}$ and $\vec{x} - \vec{y} = 3\vec{i} + 6\vec{j} - 7\vec{k}$, determine $\vec{x}$ and $\vec{y}$.

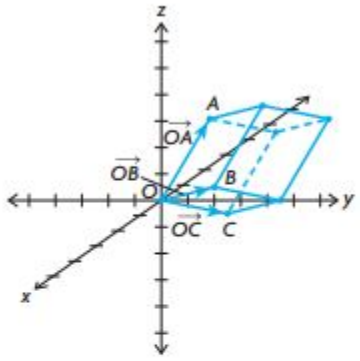
10. Given the points $A(-2, -6, 3)$ and $B(3, -4, 12)$, determine each of the following:
    a. $\left|\overrightarrow{OA}\right|$ c. $\overrightarrow{AB}$ e. $\overrightarrow{BA}$
    b. $\left|\overrightarrow{OB}\right|$ d. $\left|\overrightarrow{AB}\right|$ f. $\left|\overrightarrow{BA}\right|$

12. Given $2\vec{x} + \vec{y} - 2\vec{z} = \vec{0}$, $\vec{x} = (-1, b, c)$, $\vec{y} = (a, -2, c)$, and $\vec{z} = (-a, 6, c)$, determine the value of the unknowns.

13. A parallelepiped is determined by the vectors $\overrightarrow{OA} = (-2, 2, 5)$, $\overrightarrow{OB} = (0, 4, 1)$, and $\overrightarrow{OC} = (0, 5, -1)$.
    a. Draw a sketch of the parallelepiped formed by these vectors.
    b. Determine the coordinates of all of the vertices for the parallelepiped.
14. Given the points $A(-2, 1, 3)$ and $B(4, -1, 3)$, determine the coordinates of the point on the $x$-axis that is equidistant from these two points.

Answers to Selected Problems

**1.** **a.** $-1\vec{i} + 2\vec{j} + 4\vec{k}$
**b.** about 4.58
**2.** $\overrightarrow{OB} = (3, 4, -4)$, $|\overrightarrow{OB}| = 6.40$
**3.** 3
**5.** **a.** $(1, -3, 3)$
**b.** $(-7, -16, 8)$
**c.** $\left(-\frac{13}{2}, 2, \frac{3}{2}\right)$
**d.** $(2, 30, -13)$
**6.** **a.** $\vec{i} - 2\vec{j} + 2\vec{k}$
**b.** $3\vec{i} + 0\vec{j} + 0\vec{k}$
**c.** $9\vec{i} + 3\vec{j} - 3\vec{k}$
**d.** $-9\vec{i} - 3\vec{j} + 3\vec{k}$
**7.** **a.** about 5.10
**b.** about 1.41
**c.** about 5.39
**d.** about 11.18
**8.** $\vec{x} = \vec{i} + 4\vec{j} - \vec{k}$, $\vec{y} = -2\vec{i} - 2\vec{j} + 6\vec{k}$
**10.** **a.** 7
**b.** 13
**c.** $(5, 2, 9)$
**d.** 10.49
**e.** $(-5, -2, -9)$
**f.** 10.49
**12.** $a = \frac{2}{3}, b = 7, c = 0$
**13.** **a.**




**b.** $V_1 = (0, 0, 0)$,
$V_2 = (-2, 2, 5)$,
$V_3 = (0, 4, 1)$,
$V_4 = (0, 5, -1)$,
$V_5 = (-2, 6, 6)$,
$V_6 = (-2, 7, 4)$,
$V_7 = (0, 9, 0)$,
$V_8 = (-2, 11, 5)$
**14.** $(1, 0, 0)$