6.8a Linear Combinations and Spanning Sets

These problems taken from the Nelson Text: Pg. 341

7. Simplify each of the following linear combinations and write your answer in component form: $\vec{a} = \vec{i} - 2\vec{j}$, $\vec{b} = \vec{j} - 3\vec{k}$, and $\vec{c} = \vec{i} - 3\vec{j} + 2\vec{k}$
   a. $2(2\vec{a} - 3\vec{b} + \vec{c}) - 4(-\vec{a} + \vec{b} - \vec{c}) + (\vec{a} - \vec{c})$
   b. $\frac{1}{2}(2\vec{a} - 4\vec{b} - 8\vec{c}) - \frac{1}{3}(3\vec{a} - 6\vec{b} + 9\vec{c})$
10. Solve for a, b, and c in the following equation:
    $2(a, 3, c) + 3(c, 7, c) = (5, b + c, 15)$
11. Write the vector $(-10, -34)$ as a linear combination of the vectors $(-1, 3)$ and $(1, 5)$.

The following problems are taken from the Nelson Text: Pg. 344 (Review)

2. If $\vec{x} = 2\vec{a} - 3\vec{b} - 4\vec{c}$, $\vec{y} = -2\vec{a} + 3\vec{b} + 3\vec{c}$, and $\vec{z} = 2\vec{a} - 3\vec{b} + 5\vec{c}$, determine simplified expressions for each of the following:
   a. $2\vec{x} - 3\vec{y} + 5\vec{z}$
   b. $3(-2\vec{x} - 4\vec{y} + \vec{z}) - (2\vec{x} - \vec{y} + \vec{z}) - 2(-4\vec{x} - 5\vec{y} + \vec{z})$
3. If $X(-2, 1, 2)$ and $Y(-4, 4, 8)$ are two points in R^3, determine the following:
   a. $\overrightarrow{XY}$ and $\left|\overrightarrow{XY}\right|$
   b. The coordinates of a unit vector in the same direction as $\overrightarrow{XY}$.
4. $X(-1, 2, 6)$ and $Y(5, 5, 12)$ are two points in R^3.
   a. Determine the components of a position vector equivalent to $\overrightarrow{YX}$.
   b. Determine the components of a *unit* vector that is in the same direction as $\overrightarrow{YX}$.
5. Find the components of the unit vector with the opposite direction to that of the vector from $M(2, 3, 5)$ to $N(8, 1, 2)$.

6. A parallelogram has its sides determined by the vectors $\overrightarrow{OA} = (3, 2, -6)$ and $\overrightarrow{OB} = (-6, 6, -2)$.
   a. Determine the components of the vectors representing the diagonals.
   b. Determine the angles between the sides of the parallelogram.

11. Calculate the values of a, b, and c in the following:
   a. $2(a, b, 4) + \frac{1}{2}(6, 8, c) - 3(7, c, -4) = (-24, 3, 25)$

16. The vectors $\vec{d}$ and $\vec{e}$ are such that $|\vec{d}| = 3$ and $|\vec{e}| = 5$, and the angle between them is 30°. Determine each of the following:
   a. $|\vec{d} + \vec{e}|$ b. $|\vec{d} - \vec{e}|$

Answers to Selected Problems

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7. **a.** $14\vec{i} - 43\vec{j} + 40\vec{k}$
b. $-7\vec{i} + 23\vec{j} - 14\vec{k}$
10. $a = -2, b = 24, c = 3$
11. $(-10, -34) = 2(-1, 3) - 8(1, 5)$

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2. **a.** $20\vec{a} - 30\vec{b} + 8\vec{c}$
b. $\vec{a} - 3\vec{b} - 3\vec{c}$
3. **a.** $\overrightarrow{XY} = (-2, 3, 6)$, $|\overrightarrow{XY}| = 7$
b. $\left(-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$
4. **a.** $(-6, -3, -6)$
b. $\left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$
5. $\left(-\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$
6. **a.** $\overrightarrow{OA} + \overrightarrow{OB} = (-3, 8, -8)$, $\overrightarrow{OA} - \overrightarrow{OB} = (9, -4, -4)$
b. $\theta \doteq 84.4°$
11. **a.** $a = -3, b = 26.5, c = 10$
16. **a.** 7.74
b. 2.83