# 6.8b Linear Combinations and Spanning Sets (b)

These problems taken from the Nelson Text: Pg. 340 – 342

2. It is claimed that $\{(1, 0, 0), (0, 1, 0), (0, 0, 0)\}$ is a set of vectors spanning $R^3$. Explain why it is not possible for these vectors to span $R^3$.

3. Describe the set of vectors spanned by $(0, 1)$. Say why this is the same set as that spanned by $(0, -1)$.

4. In $R^3$, the vector $\vec{i} = (1, 0, 0)$ spans a set. Describe the set spanned by this vector. Name two other vectors that would also span the same set.

5. It is proposed that the set $\{(0, 0), (1, 0)\}$ could be used to span $R^2$. Explain why this is not possible.

6. The following is a spanning set for $R^2$:
   $\{(-1, 2), (2, -4), (-1, 1), (-3, 6), (1, 0)\}$.
   Remove three of the vectors and write down a spanning set that can be used to span $R^2$.

8. Name two sets of vectors that could be used to span the *xy*-plane in $R^3$. Show how the vectors $(-1, 2, 0)$ and $(3, 4, 0)$ could each be written as a linear combination of the vectors you have chosen.

9. a. The set of vectors $\{(1, 0, 0), (0, 1, 0)\}$ spans a set in $R^3$. Describe this set.
   b. Write the vector $(-2, 4, 0)$ as a linear combination of these vectors.
   c. Explain why it is not possible to write $(3, 5, 8)$ as a linear combination of these vectors.
   d. If the vector $(1, 1, 0)$ were added to this set, what would these three vectors span in $R^3$?

13. a. Show that the vectors $(-1, 2, 3)$, $(4, 1, -2)$, and $(-14, -1, 16)$ do not lie on the same plane.
    b. Show that the vectors $(-1, 3, 4)$, $(0, -1, 1)$, and $(-3, 14, 7)$ lie on the same plane, and show how one of the vectors can be written as a linear combination of the other two.

15. The vectors $\vec{a}$ and $\vec{b}$ span $R^2$. What values of $m$ and $n$ will make the following statement true: $(m - 2)\vec{a} = (n + 3)\vec{b}$? Explain your reasoning.

Answers to Selected Problems (*Plus a helpful example*)

**2.** It is not possible to use $\vec{0}$ in a spanning set. Therefore, the remaining vectors only span $R^2$.

**3.** The set of vectors spanned by $(0, 1)$ is $m(0, 1)$. If we let $m = -1$, then $m(0, 1) = (0, -1)$.

**4.** $\vec{i}$ spans the set $m(1, 0, 0)$. This is any vector along the $x$-axis. Examples: $(2, 0, 0), (-21, 0, 0)$.

**5.** As in question 2, it isn't possible to use $\vec{0}$ in a spanning set.

**6.** $\{(-1, 2), (-1, 1)\}, \{(2, -4), (-1, 1)\}, \{(-1, 1), (-3, 6)\}$ are all the possible spanning sets for $R^2$ with 2 vectors.

**8.** $\{(1, 0, 0), (0, 1, 0)\}$:
$(-1, 2, 0) = -1(1, 0, 0) + 2(0, 1, 0)$
$(3, 4, 0) = 3(1, 0, 0) + 4(0, 1, 0)$
$\{(1, 1, 0), (0, 1, 0)\}$
$(-1, 2, 0) = -1(1, 1, 0) + 3(0, 1, 0)$
$(3, 4, 0) = 3(1, 1, 0) + (0, 1, 0)$

**9.** **a.** It is the set of vectors in the $xy$-plane.
**b.** $-2(1, 0, 0) + 4(0, 1, 0)$
**c.** By part a., the vector is not in the $xy$-plane. There is no combination that would produce a number other than 0 for the $z$-component.
**d.** It would still only span the $xy$-plane. There would be no need for that vector.

**13.** **a.** The statement $a(-1, 2, 3) + b(4, 1, -2) = (-14, -1, 16)$ does not have a consistent solution.
**b.** $3(-1, 3, 4) - 5(0, -1, 1) = (-3, 14, 7)$

**15.** $m = 2, n = 3$; Non-parallel vectors cannot be equal, unless their magnitudes equal 0.

## *Helpful Example*

**Proving that a given set of vectors spans $R^2$**

Show that the set of vectors $\{(2, 1), (-3, -1)\}$ is a spanning set for $R^2$.

**Solution**

In order to show that the set spans $R^2$, we write the linear combination $a(2, 1) + b(-3, -1) = (x, y)$, where $(x, y)$ represents *any* vector in $R^2$. Carrying out the same procedure as in the previous example, we obtain

① $2a - 3b = x$

② $a - b = y$

Again the process of elimination will be used to solve this system of equations.

① $2a - 3b = x$

③ $2a - 2b = 2y$, after multiplying equation ② by 2

Subtracting eliminates $a$, $-3b - (-2b) = x - 2y$

Therefore, $-b = x - 2y$ or $b = -x + 2y$. By substituting this value of $b$ into equation ②, we find $a = -x + 3y$. Therefore, the solution to this system of equations is $a = -x + 3y$ and $b = -x + 2y$.

This means that, whenever we are given the components of any vector, we can find the corresponding values of $a$ and $b$ by substituting into the formula. Since the values of $x$ and $y$ are unique, the corresponding values of $a$ and $b$ are also unique. Using this formula to write $(-3, 7)$ as a linear combination of the two given vectors, we would say $x = -3$, $y = 7$ and solve for $a$ and $b$ to obtain

$a = -(-3) + 3(7) = 24$

and

$b = -(-3) + 2(7) = 17$

$24(2, 1) + 17(-3, -1) = (-3, 7)$

So the vector $(-3, 7)$ can be written as a linear combination of $(2, 1)$ and $(-3, -1)$. Therefore, the set of vectors $\{(2, 1), (-3, -1)\}$ spans $R^2$.