VECTORS

Chapter 7 –Applications of Vectors

(*Material adapted from Chapter 7 of your text*)

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MATH@TD

Chapter 7 – Applications of Vectors

Contents with suggested problems from the Nelson Textbook (Chapter 7)

Hwk Check

4 8.8b

15. The vectors $\vec{a}$ and $\vec{b}$ span R^2. What values of m and n will make the following statement true: $(m - 2)\vec{a} = (n + 3)\vec{b}$? Explain your reasoning.

Since $\text{span}\{\vec{a}, \vec{b}\} = \mathbb{R}^2$

$\Rightarrow \nexists$ scalar $k \neq 0$ s.t. $\vec{a} = k\vec{b}$ (ie $\vec{a}$ & $\vec{b}$ are not collinear)

$\Rightarrow$ in $(m-2)\vec{a} = (n+3)\vec{b}$

$m - 2 = 0$ and $n+3 = 0$

$\Rightarrow m = 2$ | $n = -3$

7.1 & 7.2 Vectors as Force and Acceleration

Both **force** and **velocity** are ‘real world’ qualities which have **magnitude** (size) and **direction**. Thus we can use the mathematics of vectors to solve “real world problems”.

Example 7.1.1

A box is being pushed along the floor by Fred and Sally. Fred pushes the box with a force of $35N\,[right]$. Sally pushes with a force of:

a) $40N\,[right]$ b) $40N\,[left]$

For a) and b) determine i) the resultant force $\vec{f_r}$ and ii) the equilibrant of the system $\vec{e}$.

a) $\vec{F} = 35N\,[r]$

$\vec{S} = 40N\,[r]$

$\vec{F}+\vec{S} = 75N\,[r]$

$\vec{e} = 75N\,[\ell]$

b) $\vec{S} = 40N\,[\ell]$

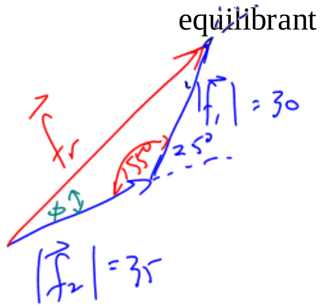
$\vec{F} = 35N\,[r]$

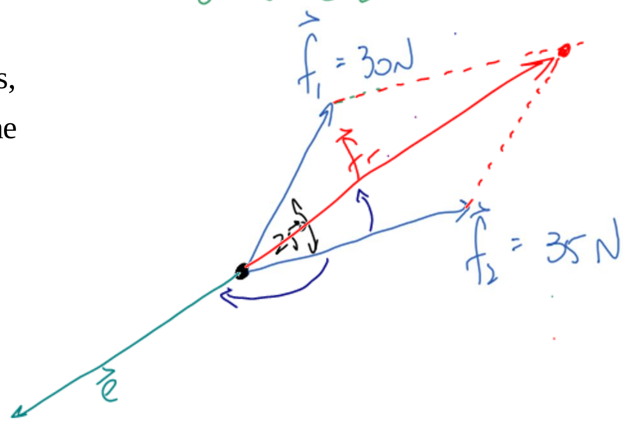
$\vec{F}+\vec{S} = 5N\,[\ell]$

$\vec{e} = 5N\,[r]$

Example 7.1.2

Given the diagram of a system of forces, determine the resultant force $\vec{f_r}$, and the equilibrant $\vec{e}$.





$$|\vec{f_r}| = \sqrt{35^2 + 30^2 - 2(35)(30)\cos(155)}$$

$$= 63.47\ N$$

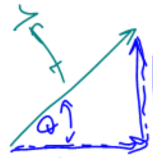
we still need direction

$$\frac{\sin\phi}{30} = \frac{\sin 155}{63.47}$$

$$\phi = \sin^{-1}\left(\frac{(30)(\sin(155))}{63.47}\right)$$

$$= 11.5°$$

$\therefore \vec{f_r} = 63.47N$, $11.5°$ from $\vec{f_2}$ $\quad \therefore \vec{e} = 63.47\ N$ $168.5°$ from $\vec{f_2}$

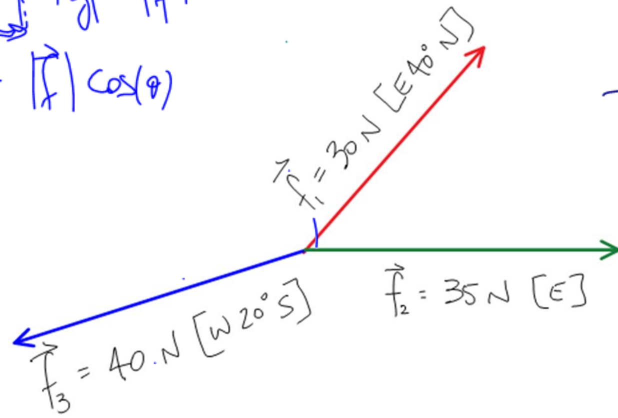


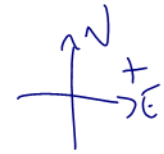
$|\vec{f_y}| = |\vec{f}| \sin(\theta)$

$|\vec{f_x}| = |\vec{f}| \cos(\theta)$

Example 7.1.3

Consider the following sketch of a system of forces:





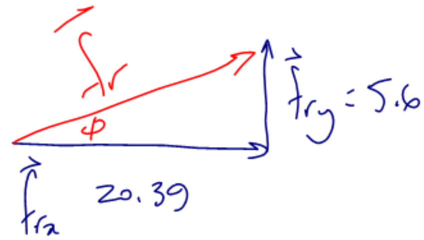
Determine $\vec{f_r}$.

We will separate all forces into their rectangular components $\vec{f_x}$, $\vec{f_y}$ (using a chart)

VECTOR	x component	y component
$\vec{f_1} = 30N$ [E 40° N]	$\vec{f_{1x}} = 30\cos(40)$ [E] $= 22.98$ [E]	$\vec{f_{1y}} = 30\sin(40)$ [N] $= 19.28$ [N]
$\vec{f_2} = 35\,N$ [E]	$\vec{f_{2x}} = 35$ [E]	$\vec{f_{2y}} = \vec{0}$
$\vec{f_3} = 40\,N$ [W 20° S]	$\vec{f_{3x}} = 40\cos(20)$ [W] $= 37.59$ [W]	$\vec{f_{3y}} = 40\sin(20)$ [S] $= 13.68$ [S]
$\vec{f_r} = ?$	$\vec{f_{rx}} = 22.98 + 35 - 37.59$ $= 20.39\,N$ [E]	$\vec{f_{ry}} = 19.28 - 13.68$ [N] $= 5.6\,N$ [N]

Now we need $\vec{f_r}$

The "resulting" forces



magnitude

$$|\vec{f}_r| = \sqrt{(|\vec{f}_{rx}|)^2 + (|\vec{f}_{ry}|)^2}$$

$$= \sqrt{(20.39)^2 + (5.6)^2}$$

$$= 21.14 \text{ N}$$

direction

$$\tan\phi = \frac{5.6}{20.39}$$

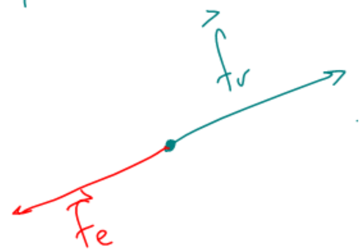
$$\Rightarrow \phi = \tan^{-1}\left(\frac{5.6}{20.39}\right)$$

$$= 15.4^\circ$$

$$\therefore \vec{f}_r = 21.14 \text{ N } [\text{E } 15.4^\circ \text{ N}]$$

Another look at Equilibrium

Recall that a system of vectors in equilibrium can be represented by two "opposite" vectors:

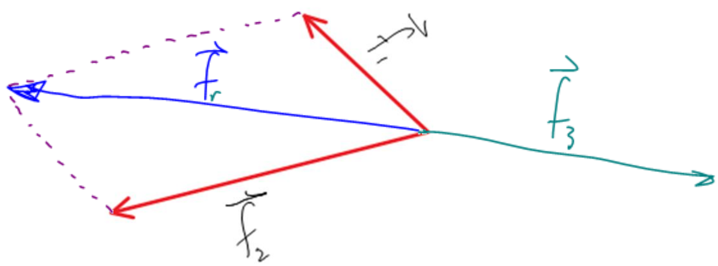


$$\vec{f}_e = -\vec{f}_r$$

$$\Rightarrow \vec{f}_e + \vec{f}_r = \vec{0}$$

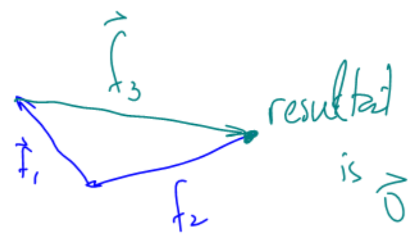
Example 7.1.4

Given that three forces $\vec{f_1}$, $\vec{f_2}$, and $\vec{f_3}$ are in equilibrium, with $\vec{f_1}$ and $\vec{f_2}$ as shown, determine $\vec{f_3}$ (as a sketch).



Note: A system in equilibrium has "no resultant"

ie $\vec{f}_1 + \vec{f}_2 + \vec{f}_3 = \vec{0}$



Class/Homework for Section 7.1

Pg. 362 – 364 #2 – 6, 8, 9, 15,
Pg. 369 – 370 #2 – 4, 6, 7, 9, 11