Hwk Check.

6. Which of the following sets of forces acting on an object could produce equilibrium?

a. 2 N, 3 N, 4 N
b. 9 N, 40 N, 41 N
c. $\sqrt{5}$ N, 6 N, 9 N
d. 9 N, 10 N, 19 N

→ No "net" force
⇒ the vectors (forces) can add to $\vec{0}$

a) $2+3>4 \Rightarrow$ ys

b) $9+40>41 \Rightarrow$ ys

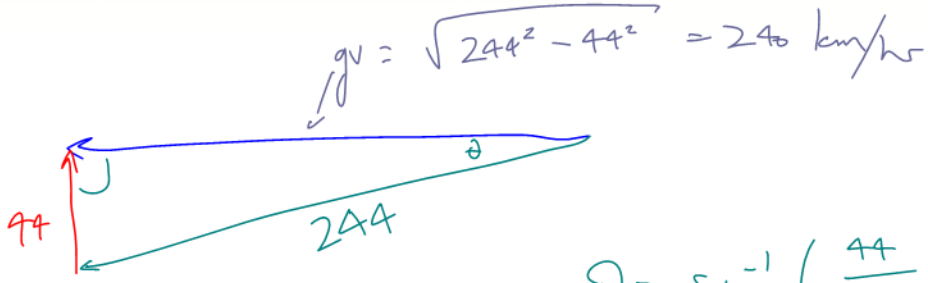
c) $\sqrt{5}+6<9 \Rightarrow$ No

d) $9+10=19 \Rightarrow$ ys

9. A small airplane has an air speed of 244 km/h. The pilot wishes to fly to a destination that is 480 km due west from the plane's present location. There is a 44 km/h wind from the south.

a. In what direction should the pilot fly in order to reach the destination?

b. How long will it take to reach the destination?



$\theta = \sin^{-1}\left(\frac{44}{244}\right)$

$= 10.4°$

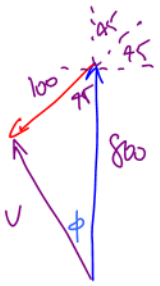
→ Direction should be [W 10.4° S]

b) $t = \frac{d}{v} = \frac{480}{240} = 2$ hours.

7. An airplane is heading due north at 800 km/h when it encounters a wind from the northeast at 100 km/h.

   a. What is the resultant velocity of the airplane?

   b. How far will the plane travel in 1 h?



$$|\vec{V}| = \sqrt{800^2 + 100^2 - 2(800)(100)\cos(45)}$$

$$= 732.7 \text{ km/hr}$$

$$\frac{\sin\phi}{100} = \frac{\sin(45)}{732.7}$$

$$\Rightarrow \quad \phi = \sin^{-1}\left(\frac{100 \cdot \sin(45)}{732.7}\right)$$

$$= 5.5°$$

$\therefore$ The velocity is

732.7 km/hr [N 5.5° W]

7.3 The Dot Product: A Geometric View

θ is NOT the angle between the vectors

Definition 7.3.1

Given vectors $\vec{a}$ and $\vec{b}$ with angle θ between them, then the Dot Product is given by:

$$\vec{a}\cdot\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos(\theta)$$

θ = angle between their directions

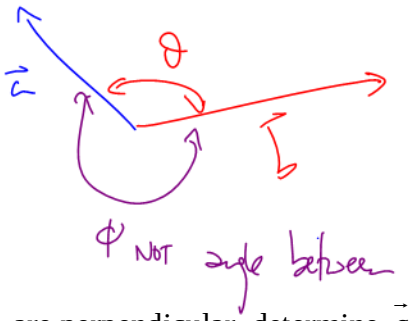
Note: $\left|\vec{a}\right|$, $\left|\vec{b}\right|$ and $\cos(\theta)$ are all just NUMBERS

$\Rightarrow$ the DOT PRODUCT IS A SCALAR

Further note that the Dot Product depends on the cosine of an angle. Thus, the Dot Product will have a sign + or −

Now, $\forall$ vectors $\vec{a}$ and $\vec{b}$, the angle θ between the vectors has the property that

for all



$0 \le \theta \le 180$

(Note: when $90 \le \theta \le 180$ $\Rightarrow \vec{a}\cdot\vec{b} < 0$)

Example 7.3.1

Given that two vectors $\vec{a}$ & $\vec{b}$ are perpendicular, determine $\vec{a}\cdot\vec{b}$

$\vec{a} \perp \vec{b} \Rightarrow$ the angle between them is 90 ($\pi/2$)

$$\Rightarrow \vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos(\pi/2)$$
$$= 0$$

Note: Given that $\vec{a}\cdot\vec{b} = 0$

It MAY be that $\vec{a} \perp \vec{b}$ OK

$\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$

Example 7.3.2

a) Given $|\vec{a}| = 5$, $|\vec{b}| = 3$ and the angle between them is $\frac{\pi}{4}$, determine $\vec{a} \cdot \vec{b}$.

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos(\theta)$

$= (5)(3)\left(\cos(\pi/4)\right) = 15\left(\frac{1}{\sqrt{2}}\right) = \frac{15\sqrt{2}}{2}$

b) Given $|\vec{c}| = 7$, determine $\vec{c} \cdot \vec{c}$. Note $\theta = 0°$

$\vec{c} \cdot \vec{c} = |\vec{c}|\,|\vec{c}| \cos(\theta)$

$= (7)(7)\cos(0) = 49 \quad (7^2)$

$\Rightarrow \forall$ vectors $\vec{a}$

$\vec{a} \cdot \vec{a} = |\vec{a}|^2$

$\Rightarrow |\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$

Algebraic Properties of the Dot Product

Given vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ and scalar k,

1) $\vec{a} \cdot \vec{b} =$ $\vec{b} \cdot \vec{a}$ (Commutative Property)

2) $\vec{a} \cdot (\vec{b} + \vec{c}) =$ $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ (Distributive Property)

3) $\vec{a} \cdot \vec{a} =$ $|\vec{a}|^2$

4) $k(\vec{a} \cdot \vec{b}) =$ $k\vec{a} \cdot \vec{b} = \vec{a} \cdot k\vec{b}$ (Associative Property of Scalar Multiplication)

No dot!

Note: $\vec{a} \cdot (\vec{b} \cdot \vec{c})$ — $(\vec{b} \cdot \vec{c})$ is a SCALAR

WE CANNOT "dot" a vector with a scalar

The dot product is between vectors

Example 7.3.3

Given $|\vec{a}| = 3$, $|\vec{b}| = 2$ and that the vectors $\vec{u} = (2\vec{a} - 3\vec{b})$ and $\vec{v} = (\vec{a} + 2\vec{b})$ are **perpendicular**, determine the angle between $\vec{a}$ and $\vec{b}$.

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$

$\cos(\theta) = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{u}\cdot\vec{v} = 0$ ← Not $\vec{0}$

$\Rightarrow (2\vec{a} - 3\vec{b})\cdot(\vec{a} + 2\vec{b}) = 0$

$\Rightarrow 2\vec{a}\cdot\vec{a} + 4\vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{a} - 6\vec{b}\cdot\vec{b} = 0$ ($3\vec{a}\cdot\vec{b}$)

$\Rightarrow 2|\vec{a}|^2 + \vec{a}\cdot\vec{b} - 6|\vec{b}|^2 = 0$

$\Rightarrow 2(3)^2 + \vec{a}\cdot\vec{b} - 6(2)^2 = 0$

$\Rightarrow \vec{a}\cdot\vec{b} = 6$

$\therefore \cos(\theta) = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{6}{(3)(2)} = 1$

$\Rightarrow \theta = \cos^{-1}(1) = 0^\circ$

$\Rightarrow \vec{a}$ & $\vec{b}$ are collinear.

Class/Homework for Section 7.3

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