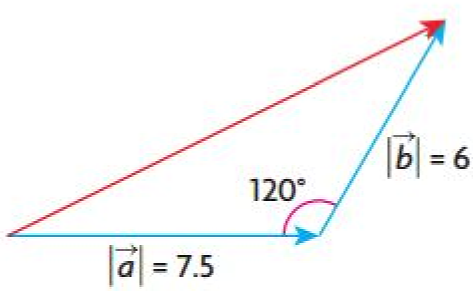
7.3 The Dot Product: Geometric View

These problems taken from the Nelson Text: Pg. 377 – 378

2. Explain why the calculation $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ is not meaningful.
3. A student writes the statement $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c}$ and then concludes that $\vec{a} = \vec{c}$. Construct a simple numerical example to show that this is not correct if the given vectors are all nonzero.
4. Why is it correct to say that if $\vec{a} = \vec{c}$, then $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c}$?
5. If two vectors $\vec{a}$ and $\vec{b}$ are unit vectors pointing in opposite directions, what is the value of $\vec{a} \cdot \vec{b}$?
6. If θ is the angle (in degrees) between the two given vectors, calculate the dot product of the vectors.
   a. $|\vec{p}| = 4$, $|\vec{q}| = 8$, $\theta = 60°$
   b. $|\vec{x}| = 2$, $|\vec{y}| = 4$, $\theta = 150°$
   c. $|\vec{a}| = 0$, $|\vec{b}| = 8$, $\theta = 100°$
7. Calculate, to the nearest degree, the angle between the given vectors.
   a. $|\vec{x}| = 8$, $|\vec{y}| = 3$, $\vec{x} \cdot \vec{y} = 12\sqrt{3}$
   d. $|\vec{p}| = 1$, $|\vec{q}| = 5$, $\vec{p} \cdot \vec{q} = -3$
   b. $|\vec{m}| = 6$, $|\vec{n}| = 6$, $\vec{m} \cdot \vec{n} = 6$
8. For the two vectors $\vec{a}$ and $\vec{b}$ whose magnitudes are shown in the diagram below, calculate the dot product.



11. The vectors $\vec{a} - 5\vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular. If $\vec{a}$ and $\vec{b}$ are unit vectors, then determine $\vec{a} \cdot \vec{b}$.

12. If $\vec{a}$ and $\vec{b}$ are any two nonzero vectors, prove each of the following to be true:

a. $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$

b. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

13. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ satisfy the relationship $\vec{a} = \vec{b} + \vec{c}$.

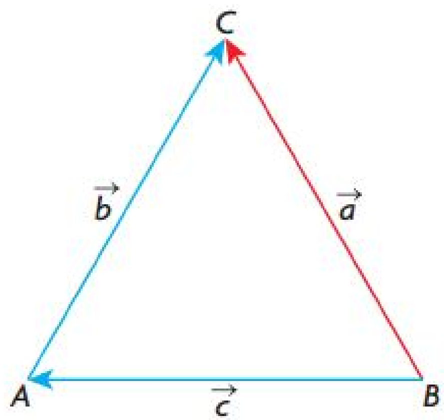
a. Show that $|\vec{a}|^2 = |\vec{b}|^2 + 2\vec{b} \cdot \vec{c} + |\vec{c}|^2$.

b. If the vectors $\vec{b}$ and $\vec{c}$ are perpendicular, how does this prove the Pythagorean theorem?

14. Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be three mutually perpendicular vectors of lengths 1, 2, and 3, respectively. Calculate the value of $(\vec{u} + \vec{v} + \vec{w}) \cdot (\vec{u} + \vec{v} + \vec{w})$.

15. Prove the identity $|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 = 2|\vec{u}|^2 + 2|\vec{v}|^2$.

16. The three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are of unit length and form the sides of equilateral triangle ABC such that $\vec{a} - \vec{b} - \vec{c} = \vec{0}$ (as shown). Determine the numerical value of $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b} + \vec{c})$.



Answers to Selected Problems

2. $\vec{a} \cdot \vec{b}$ is a scalar, and a dot product is only defined for vectors.

3. Answers may vary. For example, let $\vec{a} = \hat{i}, \vec{b} = \vec{j}, \vec{c} = -\vec{i}\ \hat{i} \cdot \vec{a} \cdot \vec{b} \cdot \vec{b} = 0$, $\vec{a} \cdot \vec{c} \cdot \vec{c} = 0$, but $\vec{a} = -\vec{c}$.

4. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = \vec{b} \cdot \vec{c}$ because $\vec{c} = \vec{a}$

5. -1

6. **a.** 16
b. -6.93
c. 0

7. **a.** 30°
b. 80°
c. 53°
d. 127°

8. 22.5

11. 1

14. 14

15.
$$\begin{aligned}|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 &= (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) \\ &\quad + (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) \\ &= |\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 + |\vec{u}|^2 \\ &\quad - 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 \\ &= 2|\vec{u}|^2 + 2|\vec{v}|^2\end{aligned}$$

16. 3