7.4 – The Dot Product: An Algebraic View

These Problems taken from the Nelson Text: Pg. 385 – 387

1. How many vectors are perpendicular to $\vec{a} = (-1, 1)$? State the components of three such vectors.
2. For each of the following pairs of vectors, calculate the dot product and, on the basis of your result, say whether the angle between the two vectors is acute, obtuse, or 90°.
   a. $\vec{a} = (-2, 1), \vec{b} = (1, 2)$
   b. $\vec{a} = (2, 3, -1), \vec{b} = (4, 3, -17)$
   c. $\vec{a} = (1, -2, 5), \vec{b} = (3, -2, -2)$
3. Give the components of a vector that is perpendicular to each of the following planes:
   a. *xy*-plane
   b. *xz*-plane
   c. *yz*-plane
6. Determine the angle, to the nearest degree, between each of the following pairs of vectors:
   a. $\vec{a} = (5, 3)$ and $\vec{b} = (-1, -2)$
   b. $\vec{a} = (-1, 4)$ and $\vec{b} = (6, -2)$
   c. $\vec{a} = (2, 2, 1)$ and $\vec{b} = (2, 1, -2)$
7. Determine k, given two vectors and the angle between them.
   a. $\vec{a} = (-1, 2, -3), \vec{b} = (-6k, -1, k), \theta = 90°$
   b. $\vec{a} = (1, 1), \vec{b} = (0, k), \theta = 45°$
9. Determine the angle, to the nearest degree, between each pair of vectors.
   a. $\vec{a} = (1 - \sqrt{2}, \sqrt{2}, -1)$ and $\vec{b} = (1, 1)$
   b. $\vec{a} = (\sqrt{2} - 1, \sqrt{2} + 1, \sqrt{2})$ and $\vec{b} = (1, 1, 1)$

10. a. For the vectors $\vec{a} = (2, p, 8)$ and $\vec{b} = (q, 4, 12)$, determine values of p and q so that the vectors are

i. collinear

ii. perpendicular

b. Are the values of p and q unique? Explain why or why not.

11. $\triangle ABC$ has vertices at $A(2, 5)$, $B(4, 11)$, and $C(-1, 6)$. Determine the angles in this triangle.

13. a. Given the vectors $\vec{p} = (-1, 3, 0)$ and $\vec{q} = (1, -5, 2)$, determine the components of a vector perpendicular to each of these vectors.

b. Given the vectors $\vec{m} = (1, 3, -4)$ and $\vec{n} = (-1, -2, 3)$, determine the components of a vector perpendicular to each of these vectors.

14. Find the value of p if the vectors $\vec{r} = (p, p, 1)$ and $\vec{s} = (p, -2, -3)$ are perpendicular to each other.

17. The vectors $\vec{x} = (-4, p, -2)$ and $\vec{y} = (-2, 3, 6)$ are such that $\cos^{-1}\left(\frac{4}{21}\right) = \theta$, where θ is the angle between $\vec{x}$ and $\vec{y}$. Determine the value(s) of p.

Answers to Selected Problems

1. Any vector of the form (c, c) is perpendicular to $\vec{a}$. Therefore, there are infinitely many vectors perpendicular to $\vec{a}$. Answers may vary. For example: $(1, 1), (2, 2), (3, 3)$.

2. **a.** 0; 90°
b. $34 > 0$; acute
c. $-3 < 0$; obtuse

3. Answer may vary. For example:
a. $(0, 0, 1)$ is perpendicular to every vector in the xy-plane.
b. $(0, 1, 0)$ is perpendicular to every vector in the xz-plane.
c. $(1, 0, 0)$ is perpendicular to every vector in the yz-plane.

6. **a.** about 148°
b. about 123°
c. about 64°
d. about 154°

7. **a.** $k = \frac{2}{3}$
b. $k \geq 0$

9. **a.** 90°
b. 30°

10. **a.** **i.** $p = \frac{8}{3}$; $q = 3$
ii. Answers may vary. For example, $p = 1, q = -50$.
b. Unique for collinear vectors; not unique for perpendicular vectors

11. $\theta_A = 90°$; $\theta_B \doteq 26.6°$; $\theta_C \doteq 63.4°$

13. **a.** Answers may vary. For example, $(3, 1, 1)$.
b. Answers may vary. For example, $(1, 1, 1)$.

14. 3 or -1

17. 4 or $-\frac{44}{65}$