7.5 Projections: Scalar and Vector

These Problems taken from the Nelson Text: Pg. 398 – 400

3. Consider two nonzero vectors, $\vec{a}$ and $\vec{b}$, that are perpendicular to each other. Explain why the scalar and vector projections of $\vec{a}$ on $\vec{b}$ must be 0 and $\vec{0}$, respectively. What are the scalar and vector projections of $\vec{b}$ on $\vec{a}$?
5. Using the formulas in this section, determine the scalar and vector projections of $\overrightarrow{OP} = (-1, 2, -5)$ on $\vec{i}$, $\vec{j}$, and $\vec{k}$. Explain how you could have arrived at the same answer without having to use the formulas.
6. a. For the vectors $\vec{p} = (3, 6, -22)$ and $\vec{q} = (-4, 5, -20)$, determine the scalar and vector projections of $\vec{p}$ on $\vec{q}$.

   b. Determine the direction angles for $\vec{p}$.
7. For each of the following, determine the scalar and vector projections of $\vec{x}$ on $\vec{y}$.

   a. $\vec{x} = (1, 1), \vec{y} = (1, -1)$

   b. $\vec{x} = (2, 2\sqrt{3}), \vec{y} = (1, 0)$

   c. $\vec{x} = (2, 5), \vec{y} = (-5, 12)$
8. a. Determine the scalar and vector projections of $\vec{a} = (-1, 2, 4)$ on each of the three axes.

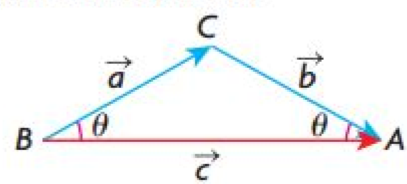
   b. What are the scalar and vector projections of $m(-1, 2, 4)$ on each of the three axes?

12. In the diagram shown, $\triangle ABC$ is an isosceles triangle where $|\vec{a}| = |\vec{b}|$.

    a. Draw the scalar projection of $\vec{a}$ on $\vec{c}$.

    b. Relocate $\vec{b}$, and draw the scalar projection of $\vec{b}$ on $\vec{c}$.

    c. Explain why the scalar projection of $\vec{a}$ on $\vec{c}$ is the same as the scalar projection of $\vec{b}$ on $\vec{c}$.



15. a. If α, β, and γ represent the direction angles for vector $\overrightarrow{OP}$, prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

b. Determine the coordinates of a vector $\overrightarrow{OP}$ that makes an angle of 30° with the y-axis, 60° with the z-axis, and 90° with the x-axis.

Answers to Selected Problems

3. You are projecting $\vec{a}$ onto the tail of $\vec{b}$, which is a point with magnitude 0. Therefore, it is $\vec{0}$; the projections of $\vec{b}$

5. scalar projection of $\vec{a}$ on $\vec{i} = -1$, vector projection of $\vec{a}$ on $\vec{i} = -\vec{i}$, scalar projection of $\vec{a}$ on $\vec{j} = 2$, vector projection of $\vec{a}$ on $\vec{j} = 2\vec{j}$, scalar projection of $\vec{a}$ on $\vec{k} = -5$, vector projection of $\vec{a}$ on $\vec{k} = -5\vec{k}$; Without having to use formulae, a projection of $(-1, 2, 5)$ on $\vec{i}, \vec{j}$, or $\vec{k}$ is the same as a projection of $(-1, 0, 0)$ on $\vec{i}$, $(0, 2, 0)$ on $\vec{j}$, and $(0, 0, 5)$ on $\vec{k}$, which intuitively yields the same result.

6. **a.** scalar projection: $\frac{\vec{p} \cdot \vec{q}}{|\vec{q}|} = \frac{458}{21}$,

vector projection: $\frac{458}{441}(-4, 5, -20)$

b. about 82.5°, about 74.9°, about 163.0°

7. **a.** scalar projection: 0, vector projection: $\vec{0}$

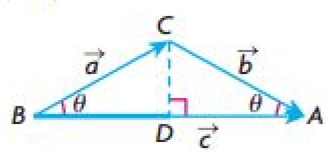
b. scalar projection: 2, vector projection: $2\vec{i}$

c. scalar projection: $\frac{50}{13}$, vector projection: $\frac{50}{169}(-5, 12)$

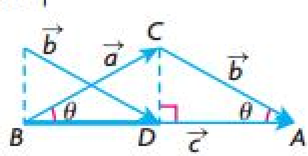
8. **a.** The scalar projection of $\vec{a}$ on the x-axis $(X, 0, 0)$ is -1; The vector projection of $\vec{a}$ on the x-axis is $-\vec{i}$; The scalar projection of $\vec{a}$ on the y-axis $(0, Y, 0)$ is 2; The vector projection of $\vec{a}$ on the y-axis is $2\vec{j}$; The scalar projection of $\vec{a}$ on the z-axis $(0, 0, Z)$ is 4; The vector projection of $\vec{a}$ on the z-axis is $4\vec{k}$.

b. The scalar projection of $m\,\vec{a}$ on the x-axis $(X, 0, 0)$ is $-m$; The vector projection of $m\,\vec{a}$ on the x-axis is $-m\vec{i}$; The scalar projection of $m\,\vec{a}$ on the y-axis $(0, Y, 0)$ is $2m$; The vector projection $m\,\vec{a}$ on the y-axis $(0, Y, 0)$ is $2m\vec{j}$; The scalar projection of $m\,\vec{a}$ on the z-axis $(0, 0, Z)$ is $4m$; The vector projection of $m\,\vec{a}$ on the z-axis is $4m\vec{k}$.

12. **a.** $|\overrightarrow{BD}|$



b. $|\overrightarrow{BD}|$



c. In an isosceles triangle, CD is a median and a right bisector of BA.

d. Yes