VECTORS

Chapter 8 –Equations of Lines and Planes

(*Material adapted from Chapter 8 of your text*)

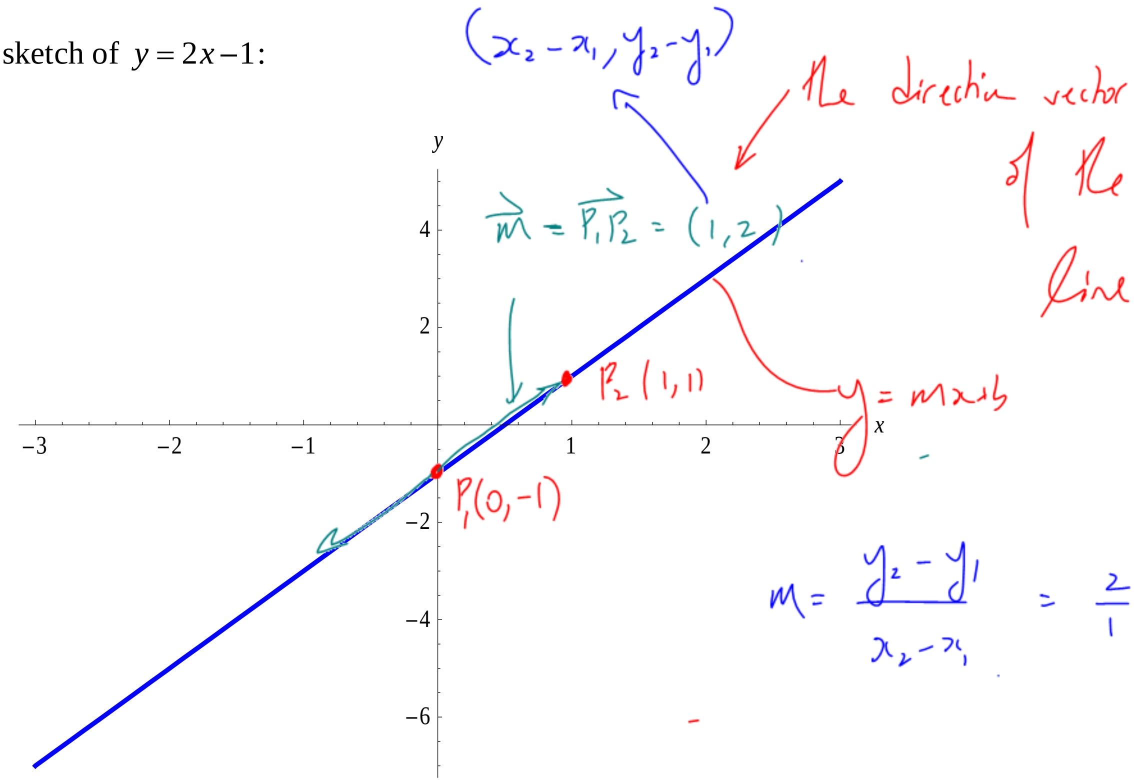
AΩ∞Ω

MATH@TD

8.1 Vector and Parametric Equations of Lines (1)

Recall: A line is a **set of points** $\{(x,y) \mid y = mx+b\}$ where $y = mx+b$ is a functional relationship between domain and range values.

Consider the sketch of $y = 2x - 1$:



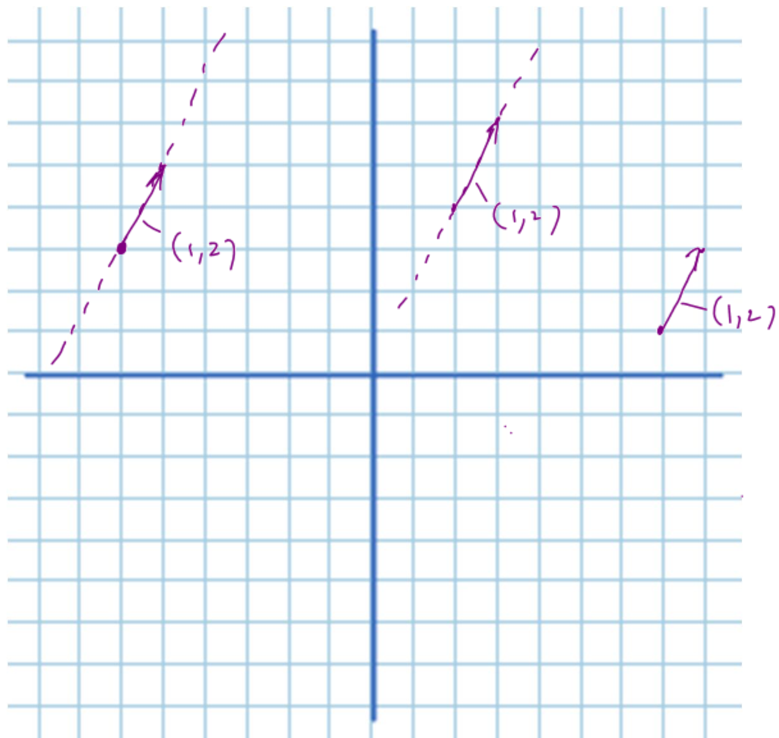
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{1} = \frac{\text{"}y\text{"}}{\text{"}x\text{"}}$$

$$\vec{m} = (x_2 - x_1, y_2 - y_1)$$

$$= (1, 2)$$

Problem: Consider a line with a direction vector $\vec{m} = (1, 2)$

we need 2 bits of info to define a line



we need a slope / direction
point / point

Eqns

Scalar: $y = mx + b$ 2 bits of info. (2 pts or slope and pt)

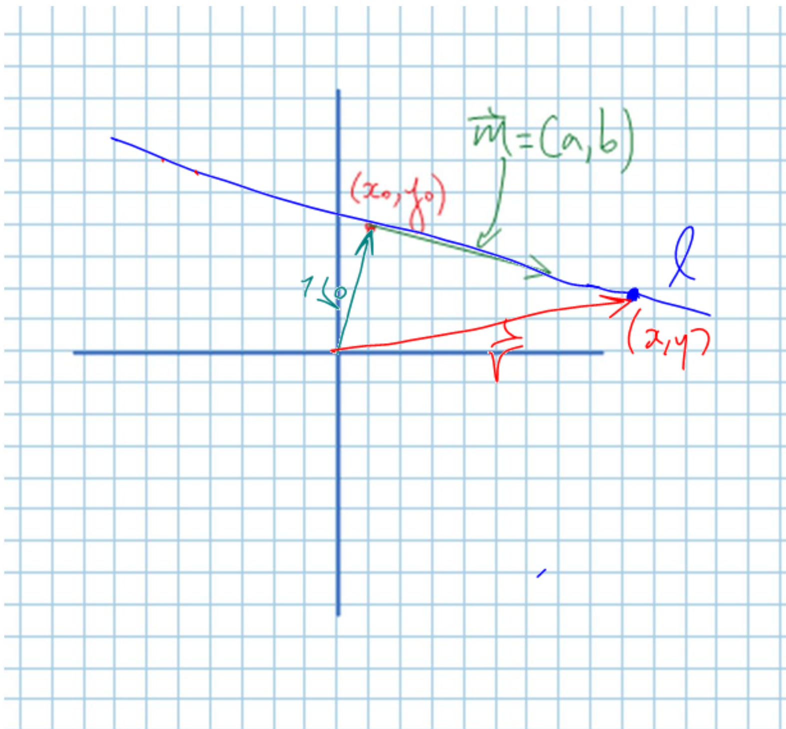
Vector: $\boxed{}$

2 bits of info
direction vector (like a slope)
& a point
(or 2 points)

we call $\vec{m} = (a, b)$ a direction vector with direction numbers $a + b$.

Vector Equation of a Line

Consider the sketch of a line through the **known** point (x_0, y_0) with **direction vector** $\vec{m} = (a,b)$.



$\vec{r} = \vec{r}_0 + t\vec{m}$

where t is a scalar $(t \in \mathbb{R})$

vector eqn $\quad \vec{r} = \vec{r}_0 + t\vec{m}$

$(x,y) = (x_0,y_0) + t(a,b)$

the vector eqn of a line in $\mathbb{R}^2$.

Example 8.1.1

point

Determine a vector equation of the line through $A(-1,4)$ with direction $\vec{m}=(4,1)$.

$$\vec{r} = \vec{r_0} + t\vec{m}$$

$$\Rightarrow \boxed{\vec{r} = (-1,4) + t(4,1)}$$

We can also write this as:

$$(x,y) = (-1,4) + t(4,1)$$

$$(x,y) = (-1,4) + (4t,t)$$

$$(x,y) = (-1+4t,\ 4+t)$$

$$\boxed{\begin{aligned} x &= -1+4t \\ y &= 4+t \end{aligned}}$$

parametric eqns of a line in $\mathbb{R}^2$

Note:

Q. Is the point $B(5,9)$ on our line?

Consider $5 = -1+4t$

$\Rightarrow t = 3/2$

sub into $y = 4+t$

$\Rightarrow y = 4 + 3/2 \neq 9$

$\therefore$ No $B(5,9)$ is not on our line

Q. Is the vector equation of a line **unique**?

No! 2 reasons ① $\vec{m}$ can be scaled

eg $\vec{m}=(4,1)$ then $\hat{m} = (-2,-\frac{1}{2})$

② we can use any known pt on the

$\vec{r} = \vec{r_0} + t\vec{m}$

we need
- a known pt
- a direction

Example 8.1.2

Obtain a vector equation (and parametric equations) for the line passing through the points $A(2,5)$ and $B(-1,2)$.

$\vec{m} = \vec{AB} = (-1-2, 2-5) = (-3,-3)$

we need a point: choose $A(2,5)$

vector: $\vec{r} = (2,5) + t(-3,-3)$

parametric $x = 2 - 3t$

$y = 5 - 3t$

Example 8.1.3

Determine vector and parametric equations for the line through $A(-1,3)$ and which is perpendicular to the line with vector equation $\underbrace{\vec{r} = \overset{r_0}{(2,1)} + \overset{t}{t}\overset{m}{(-2,3)}}$.

$\vec{m} = (-2,3)$ $\quad \vec{m}_\perp = (3,2)$

(or $\vec{m}_\perp = (-3,-2)$)

$y = -\frac{3}{2}x + b$

$m = -\frac{3}{2}$

$m_\perp = +\frac{2}{3}$

vector $\vec{r}_\perp = (-1,3) + t(3,2)$

parametric: $x = -1 + 3t$

$y = 3 + 2t$

Class/Homework for Section 8.1

Pg. 432 Investigation (smile as you do it)

Pg. 433 – 434 #1 – 3, 5, 6, 9 – 12