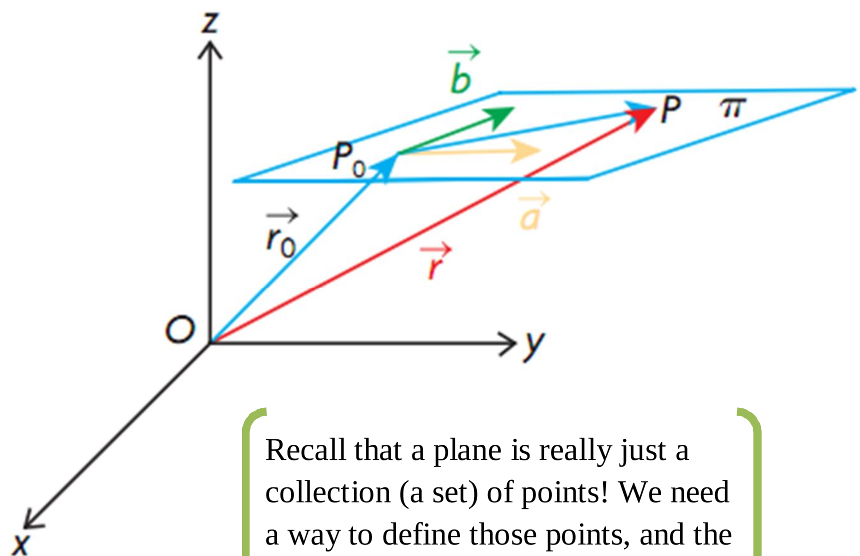
8.4 Vector and Parametric Equations of Planes

Here we will be working in $\mathbb{R}^3$. Recall that any **two non-collinear vectors will span a plane**. Thus, we can say that a **plane**, which we might call π, can be "defined" by **two direction vectors**. Consider the sketch (from your text):



Recall that a plane is really just a collection (a set) of points! We need a way to define those points, and the way we normally use is algebraic equations!

Vector and Parametric Equations of a Plane

Given a **known point** $P_0(x_0, y_0, z_0)$,

and **two direction vectors** $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$,

and **two scalars**, *s* and *t* (both real numbers), then

Vector Equation

Parametric Equations

Q. What about symmetric equations of a plane?

Example 8.4.1

From your text: Pg. 459 #1

State which of the following equations define lines and which define planes. Explain how you made your decision.

a. $\vec{r} = (1, 2, 3) + s(1, 1, 0) + t(3, 4, -6), s, t \in \mathbf{R}$

b. $\vec{r} = (-2, 3, 0) + m(3, 4, 7),\ m \in \mathbf{R}$

c. $x = -3 - t, y = 5, z = 4 + t, t \in \mathbf{R}$

d. $\vec{r} = m(4, -1, 2) + t(4, -1, 5), m, t \in \mathbf{R}$

Example 8.4.2

From your text: Pg. 459 #5

Explain why the equation $\vec{r} = (-1, 0, -1) + s(2, 3, -4) + t(4, 6, -8)$ does not describe a plane.

Example 8.4.3

Determine vector and parametric equations of a plane containing the points $A(1,2,0)$, $B(3,-2,1)$ and $C(0,2,1)$.

Example 8.4.4

From your text: Pg. 460 #15

The plane with equation $\vec{r} = (1, 2, 3) + m(1, 2, 5) + n(1, -1, 3)$ intersects the y- and z-axes at the points A and B, respectively. Determine the equation of the line that contains these two points.

Class/Homework for Section 8.4

Read Example 4 on Pg. 458
Pg. 459 – 460 #3, 4, 6, 7 (beautiful), 9 – 11, 13