8.4 Vector and Parametric Equations of Planes

These problems taken from the Nelson Text: Pg. 459 – 460

3. A plane has $x = 2m$, $y = -3m + 5n$, $z = -1 - 3m - 2n$, $m, n \in \mathbf{R}$, as its parametric equations.
   a. By inspection, identify the coordinates of a point that is on this plane.
   b. What are the direction vectors for this plane?
   c. What point corresponds to the parameter values of $m = -1$ and $n = -4$?
   d. What are the parametric values corresponding to the point $A(0, 15, -7)$?
   e. Using your answer for part d., explain why the point $B(0, 15, -8)$ cannot be on this plane.

4. A plane passes through the points $P(-2, 3, 1)$, $Q(-2, 3, 2)$, and $R(1, 0, 1)$.
   a. Using $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ as direction vectors, write a vector equation for this plane.
   b. Using $\overrightarrow{QR}$ and one other direction vector, write a second vector equation for this plane.

6. Determine vector equations and the corresponding parametric equations of each plane.
   a. the plane with direction vectors $\vec{a} = (4, 1, 0)$ and $\vec{b} = (3, 4, -1)$, passing through the point $A(-1, 2, 7)$
   b. the plane passing through the points $A(1, 0, 0)$, $B(0, 1, 0)$, and $C(0, 0, 1)$
   c. the plane passing through points $A(1, 1, 0)$ and $B(4, 5, -6)$, with direction vector $\vec{a} = (7, 1, 2)$

7. a. Determine parameters corresponding to the point $P(5, 3, 2)$, where P is a point on the plane with equation $\pi : \vec{r} = (2, 0, 1) + s(4, 2, -1) + t(-1, 1, 2)$, $s, t \in \mathbf{R}$.
   b. Show that the point $A(0, 5, -4)$ does not lie on π.

9. Determine the coordinates of the point where the plane with equation $\vec{r} = (4, 1, 6) + s(11, -1, 3) + t(-7, 2, -2), s, t \in \mathbf{R}$, crosses the z-axis.

10. Determine the equation of the plane that contains the point $P(-1, 2, 1)$ and the line $\vec{r} = (2, 1, 3) + s(4, 1, 5), s \in \mathbf{R}$.

11. Determine the equation of the plane that contains the point $A(-2, 2, 3)$ and the line $\vec{r} = m(2, -1, 7), m \in \mathbf{R}$.

Answers

3. **a.** $(0, 0, -1)$
b. $(2, -3, -3)$ and $(0, 5, -2)$
c. $(-2, -17, 10)$
d. $m = 0$ and $n = 3$
e. For the point $B(0, 15, -8)$, the first two parametric equations are the same, yielding $m = 0$ and $n = 3$; however, the third equation would then give:
$-8 = -1 - 3m - 2n$
$-8 = -1 - 3(0) - 2(3)$
$-8 = -7$
which is not true. So, there can be no solution.

9. $(0, 0, 5)$

10. $\vec{r} = (2, 1, 3) + s(4, 1, 5) + t(3, -1, 2), t, s \in \mathbf{R}$

11. $\vec{r} = m(2, -1, 7) + n(-2, 2, 3), m, n \in \mathbf{R}$

4. **a.** $\vec{r} = (-2, 3, 1) + t(0, 0, 1) + s(3, -3, 0), t, s \in \mathbf{R}$
b. $\vec{r} = (-2, 3, -2) + t(0, 0, 1) + s(3, -3, -1), t, s \in \mathbf{R}$

6. **a.** $\vec{r} = (-1, 2, 7) + t(4, 1, 0) + s(3, 4, -1), t, s \in \mathbf{R}$;
$x = -1 + 4t + 3s,$
$y = 2 + t + 4s,$
$z = 7 - s, t, s \in \mathbf{R}$
b. $\vec{r} = (1, 0, 0) + t(-1, 1, 0) + s(-1, 0, 1), t, s \in \mathbf{R}$;
$x = 1 - t - s,$
$y = t,$
$z = s, t, s \in \mathbf{R}$
c. $\vec{r} = (1, 1, 0) + t(3, 4, -6) + s(7, 1, 2), t, s \in \mathbf{R}$;
$x = 1 + 3t + 7s,$
$y = 1 + 4t + s,$
$z = -6t + 2s, t, s \in \mathbf{R}$

7. **a.** $s = 1$ and $t = 1$
b. $(0, 5, -4) = (2, 0, 1) + s(4, 2, -1) + t(-1, 1, 2)$ gives the following parametric equations:
$0 = 2 + 4s + t \Rightarrow t = 2 + 4s$
$5 = 2s + t$
$5 = 2s + (2 + 4s)$
$3 = 6s$
$\frac{1}{2} = s$
$t = 2 + 4\left(\frac{1}{4}\right)$
$t = 2 + 2 = 4$
The third equation then says:
$-4 = 1 - s + 2t$
$-4 = 1 - \frac{1}{2} + 2(4)$
$-4 = \frac{17}{2}$, which is a false statement. So, the point $A(0, 5, -4)$ is not on the plane.