8.5 The Cartesian Equation of a Plane in R^3

These problems taken from the Nelson Text: Pg. 468 – 469

1. A plane is defined by the equation $x - 7y - 18z = 0$.
   a. What is a normal vector to this plane?
   b. Explain how you know that this plane passes through the origin.
   c. Write the coordinates of three points on this plane.
2. A plane is defined by the equation $2x - 5y = 0$.
   a. What is a normal vector to this plane?
   b. Explain how you know that this plane passes through the origin.
   c. Write the coordinates of three points on this plane.

5. A plane is determined by a normal, $\vec{n} = (1, 7, 5)$, and contains the point $P(-3, 3, 5)$. Determine a Cartesian equation for this plane using the *two* methods
7. Determine the Cartesian equation of the plane that contains the points $A(-2, 3, 1)$, $B(3, 4, 5)$, and $C(1, 1, 0)$.
8. The line with vector equation $\vec{r} = (2, 0, 1) + s(-4, 5, 5)$, $s \in \mathbf{R}$, lies on the plane π, as does the point $P(1, 3, 0)$. Determine the Cartesian equation of π.
9. Determine unit vectors that are normal to each of the following planes:
   a. $2x + 2y - z - 1 = 0$
   b. $4x - 3y + z - 3 = 0$
10. A plane contains the point $A(2, 2, -1)$ and the line $\vec{r} = (1, 1, 5) + s(2, 1, 3)$, $s \in \mathbf{R}$. Determine the Cartesian equation of this plane.
11. Determine the Cartesian equation of the plane containing the point $(-1, 1, 0)$ and perpendicular to the line joining the points $(1, 2, 1)$ and $(3, -2, 0)$.
15. Determine the Cartesian equation of the plane that passes through the points $(1, 4, 5)$ and $(3, 2, 1)$ and is perpendicular to the plane $2x - y + z - 1 = 0$.

Answers on the back

1. a. $\vec{n} = (A, B, C) = (1, -7, -18)$
   b. In the Cartesian equation:
   $Ax + By + Cz + D = 0$
   If $D = 0$, the plane passes through the origin.
   c. $(0, 0, 0), (11, -1, 1), (-11, 1, -1)$
2. a. $\vec{n} = (A, B, C) = (2, -5, 0)$
   b. In the Cartesian equation: $D = 0$. So, the plane passes through the origin.
   c. $(0, 0, 0), (5, 2, 0)\ (5, 2, 1)$
5. *Method 1:* Let $A(x, y, z)$ be a point on the plane. Then,
   $\overrightarrow{PA} = (x + 3, y - 3, z - 5)$ is a vector on the plane.
   $\vec{n} \cdot \overrightarrow{PA} = 0$
   $(x + 3) + 7(y - 3) + 5(z - 5) = 0$
   $x + 7y + 5z - 43 = 0.$
   Method 2: $\vec{n} = (1, 7, 5)$ so the Cartesian equation is
   $x + 7y + 5z + D = 0$
   We know the point $(-3, 3, 5)$ is on the plane and must satisfy the equation, so
   $(-3) + 7(3) + 5(5) + D = 0$
   $43 + D = 0$
   $D = -43$
7. $7x + 17y - 13z - 24 = 0$
8. $20x + 9y + 7z - 47 = 0$
9. a. $\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$
   b. $\left(\frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right)$
   c. $\left(\frac{3}{13}, -\frac{4}{13}, \frac{12}{13}\right)$
10. $21x - 15y - z - 1 = 0$
11. $2x - 4y - z + 6 = 0$
15. $3x + 5y - z - 18 = 0$