VECTORS

Chapter 9 –Points Lines and Planes

(*Material adapted from Chapter 9 of your text*)

A∞Ω

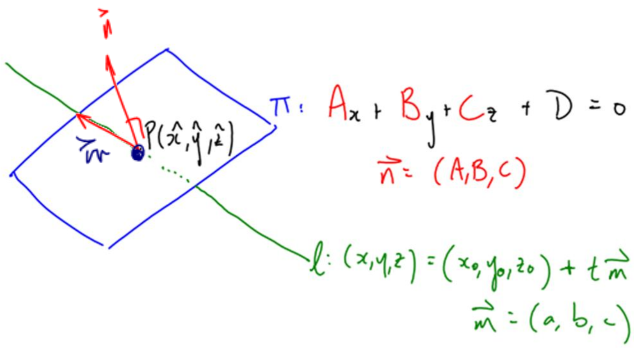
MATH@TD

9.1 Intersecting Lines and Planes

Intersecting Lines with Planes

There are three possibilities. Consider the sketches:

1)



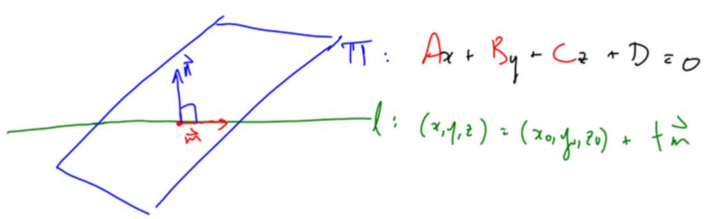
Note

$\vec{n}\cdot\vec{m} \neq 0$

$\Rightarrow$ l is not in the plane

$\Rightarrow$ l cuts the plane at one point only $P(\hat{x},\hat{y},\hat{z})$

2)



"l is coincident" to π

$\therefore$ $\vec{n} \perp \vec{m}$

$\Rightarrow \vec{n}\cdot\vec{m} = 0$

l is in the plane

l in the plane

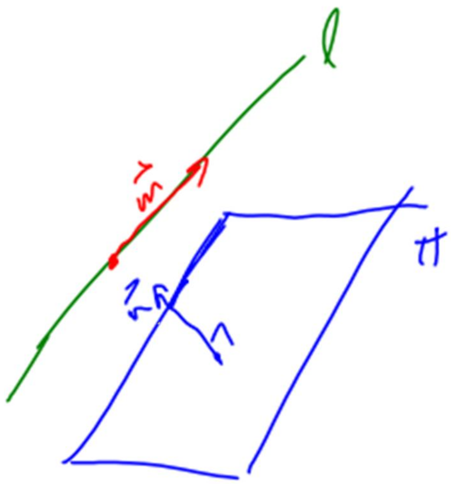
$\Rightarrow$ every point on l are in the plane

Note: $P_0(x_0, y_0, z_0)$ on l will satisfy the eqn for π

$\Rightarrow$ ∞'ly many pts of intersection

3)

ℓ & π are non-coincident



$\vec{n} \cdot \vec{m} = 0$

No point on ℓ satisfies π, particularly $P_0(x_0, y_0, z_0)$

ℓ is not in π

Further ℓ never intersects π

Example 9.1.1

Determine any points of intersection between

$l: \ (x, y, z) = (1, 2, 3) + t(1, -2, 5)$ $\qquad \vec{m} = (1, -2, 5)$ $\qquad P_0(1, 2, 3)$

$\pi: \ 2x + y - z - 21 = 0$ $\qquad \vec{n} = (2, 1, -1)$

$\vec{n} \cdot \vec{m} = (2, 1, -1) \cdot (1, -2, 5) = -5 \neq 0$

$\therefore$ we have a unique pt of intersection $P(\hat{x}, \hat{y}, \hat{z})$

Parametric

$x = 1 + t$

$y = 2 - 2t$ $\qquad$ Sub into π $\qquad 2(1+t) + (2-2t) - (3+5t) - 21 = 0$

$2 + 2t + 2 - 2t - 3 - 5t - 21 = 0$

$z = 3 + 5t$ $\qquad -5t = 20$

$\Rightarrow t = -4$

$\therefore$ Our point of intersection is $P(-3, 10, -17)$

Example 9.1.2

Determine any points of intersection of:

$l: \ x = 2 - t, \ y = 3 + 2t, \ z = -1 + t$ $\qquad \vec{m} = (-1, 2, 1) \qquad P_0(2, 3, -1)$

$\pi: \ 3x + y + z + 5 = 0$ $\qquad \vec{n} = (3, 1, 1)$

~~Read Examples 1,2 and 3 on pages 489 – 491 for different methods~~

$\vec{n} \cdot \vec{m} = (3,1,1) \cdot (-1,2,1)$

$= 0$

$\Rightarrow$ either ∞'ly many sol'ns (POI)

or no sol'n

Test $P_0(2,3,-1)$ in π

$3(2) + (3) + (-1) + 5$

$= 13 \neq 0 \quad \therefore P_0$ is NOT in π

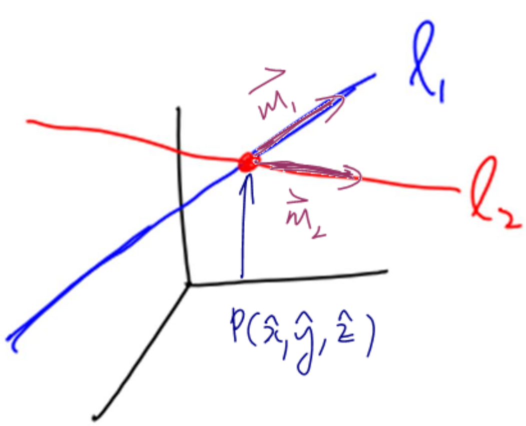
$\therefore$ No POI.

Intersecting Lines with Lines

You have found the intersection of lines in $\mathbb{R}^2$ many times in the past (using Substitution or Elimination for example). So we will work in $\mathbb{R}^3$ to keep things interesting. In $\mathbb{R}^3$ there are **four** possibilities for intersecting lines: two for having an intersection, and two when the lines do not intersect.

Consider the sketches:

1)



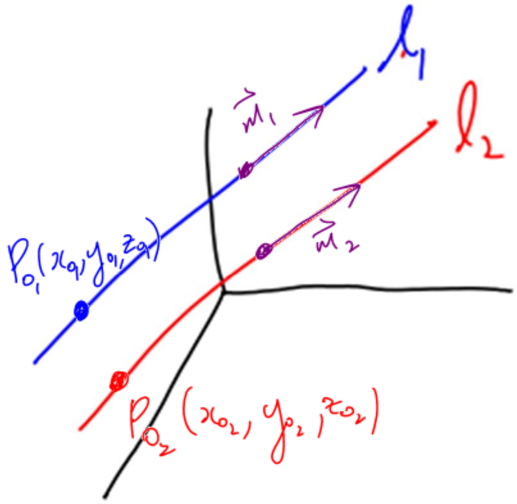
l_1 & l_2 intersect at $P(\hat{x}, \hat{y}, \hat{z})$

note

$\vec{m}_1 \neq k\,\vec{m}_2$, k is a scalar

($\vec{m}_1$ & $\vec{m}_2$ are in different direction)

2)



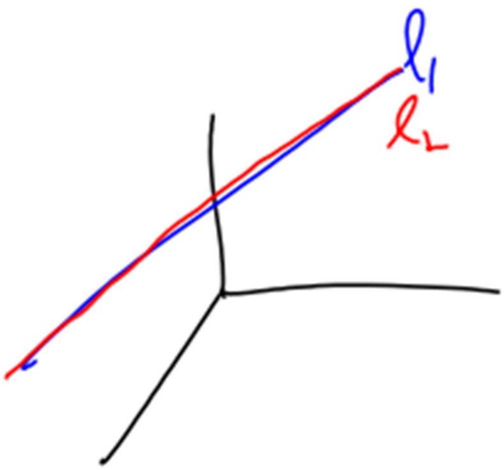
ℓ_1 & ℓ_2 do not intersect

Note $\vec{m}_1 = k\vec{m}_2$ for some k scalar

ℓ_1 & ℓ_2 are not coincident

$\Rightarrow$ they share no points

3)



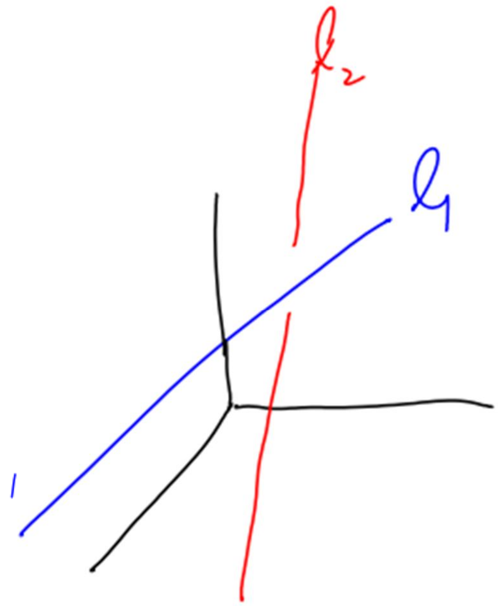
ℓ_1 & ℓ_2 are coincident

$\Longrightarrow$ ∞'ly many pts of intersection

$\vec{m}_1 = k\vec{m}_2$

(i.e., direction vectors are collinear)

4)



$\vec{m}_1 \neq k\vec{m}_2$

($\vec{m}_1$ & $\vec{m}_2$ are not collinear)

But they share no point of intersection

We call ℓ_1 & ℓ_2

SKEW LINES

Class/Homework for Section 9.1

READ ex. 4, 5, 6 Pg. 492 - 495
Pg. 496 – 498 #1, 2, 4 – 9, 11, 12, 15 (beautiful)