9.2 Systems of Linear Equations

(SOLVE)

To solve Systems of Linear Equations we will use the method of ***Elimination*** first learned (usually) in Grade 10, but we will extend the ideas to techniques required for systems in $\mathbb{R}^3$. Before getting to those techniques, it may be useful to recall what is meant by "linear equation".

linear means no term is of order greater than 1

eg

$3x + 2y - 5z - 9 = 0$ linear

$2x - 3y + \sin(z) + 3 = 0$ not linear — NOT linear "in z"

$2x + 3xy - 5z + 2 = 0$ not linear

order 2

Solving a System of Linear Equations

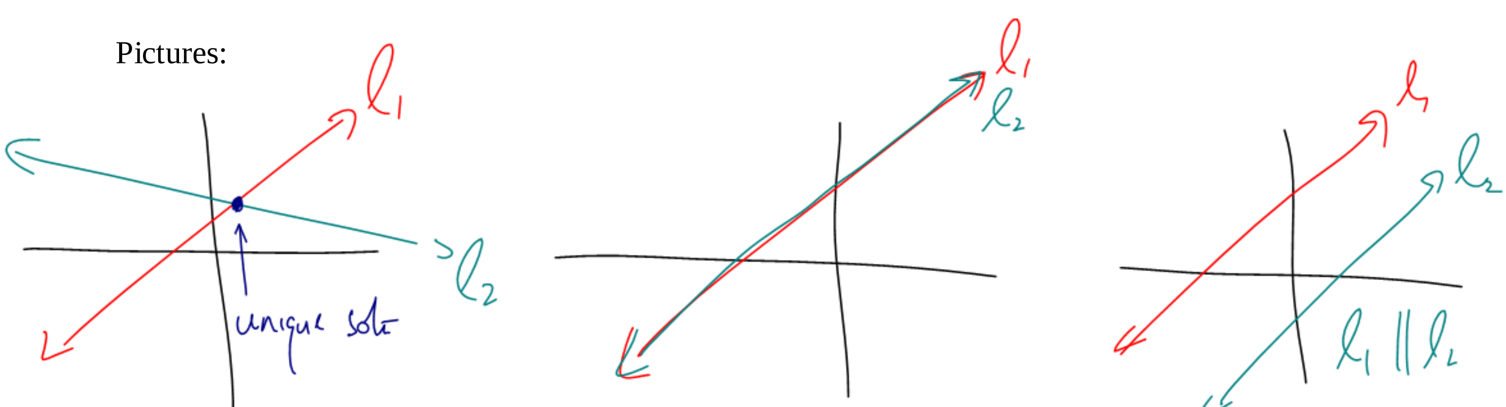
Consider the system $\left(\text{in } \mathbb{R}^2\right)$

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$

Note: This system (two equations in 2 unknowns) is either: consistent (has solns) OR

inconsistent no soln

Pictures:



consistent

consistent w/ ∞'ly many soln

inconsistent (no solns)

Example 9.2.1

Solve the system

$$3x+2y=5 \quad ①$$
$$-6x-4y=-10 \quad ②$$

Note: We will be using what we call **Elementary Row Operations** to solve our systems of equations. An **ERO** allows us to construct an **equivalent system** which is "easy" to solve.

ERO's are:

- Interchanging rows
- Multiplying/Dividing rows by a constant
- Adding/Subtracting one row from another

① ×2 $\quad 6x+4y=10 \quad ③$

② $\quad -6x-4y=-10 \quad ②$

③ + ② $\quad 0x+0y=0$

$\Rightarrow 0=0$ TRUE

Consistent system

Grade 10. soln is

$3x+2y=5$

$$\boxed{y=-\frac{3}{2}x+\frac{5}{2}}$$

We want to find a vector eqn!

We note that we have 1 eqn w/ 2 unknowns

$\Rightarrow$ one of our unknowns will be free

We want to characterize the free variable with a parameter.

Set $x=t$, then $y=-\frac{3}{2}t+\frac{5}{2}$

$\therefore$ our soln is

$(x,y)=(t, -3/2\,t+5/2)$

$x=t$

$y=-3/2\,t+5/2$

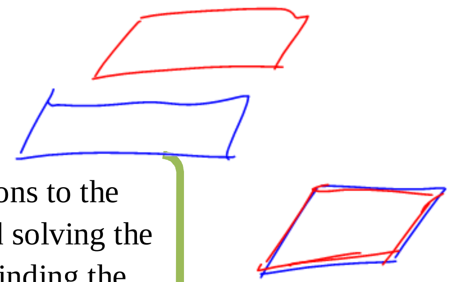
$(x,y)=(0,5/2)+t(1,-3/2)$

ex $f(x) = -2(x-1)^2 + 3$

Definition 9.2.1 (*note that this is a basic definition*)

A **parameter** is a measurable factor which defines a “particular” mathematical object.

For example, in the function $f(x) = a(x-h)^2 + k$,

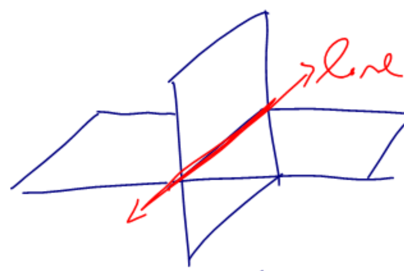


Example 9.2.2

Solve the system in $\mathbb{R}^3$

$$2x + y - 2z = 1 \quad (1)$$
$$x + 2y - 5z = 2 \quad (2)$$

Note that the two equations to the left represent **planes** and solving the system is equivalent to finding the intersection of the two planes.



$2x + y - 2z = 1 \quad (1)$

(2) × 2 $\quad 2x + 4y - 10z = 4 \quad (3)$

(3) − (1) $\quad 3y - 8z = 3$ this is as 'good' as we can get
we cannot eliminate another variable

1 eq'n − 2 unknowns
⇒ 1 free variable
⇒ parameterize

Let $y = t$

$\Rightarrow z = \frac{3}{8}t - \frac{3}{8}$ sub into (2)

$x + 2(t) - 5\left(\frac{3}{8}t - \frac{3}{8}\right) = 2$

$\Rightarrow x = -\frac{1}{8}t + \frac{1}{8}$

∴ our sol'n is

$x = -\frac{1}{8}t + \frac{1}{8}, \quad y = t, \quad z = \frac{3}{8}t - \frac{3}{8}$

parametric eq'n of a line in $\mathbb{R}^3$

Jeremy needs a hug.

Example 9.2.3

Solve the system

$2x + y - z = 6$ ①

$x - y + 2z = -1$ ②

$3x + 2y + 3z = 5$ ③

Goal: Using **ERO's** we want to construct an equivalent system which looks like:

$ax + by + cz = d$

$0x + ey + fz = g$

$0x + 0y + hz = i$

② ⟺ ①

$x - y + 2z = -1$ ①

$2x + y - z = 6$ ②

$3x + 2y + 3z = 5$ ③

$x - y + 2z = -1$ ①

② − 2① $0x + 3y - 5z = 8$ ④

③ − 3① $0x + 5y - 3z = 8$ ⑤

$x - y + 2z = -1$ ①

④×5 $0x + 15y - 25z = 40$ ⑥

⑤×3 $0x + 15y - 9z = 24$ ⑦

Notes: 1) If after using ERO's our system has a row which looks like:

$0x + 0y + 0z = 0$

⇒ consistent system w/ ∞'ly many sol'n

2) If after using ERO's we have a row which looks like:

$0x + 0y + 0z = b,\ b \neq 0$

inconsistent system

$x - y + 2z = -1$ ① r_1

$0x + 15y - 25z = 40$ ⑥ r_2

⑦ − ⑥ $0x + 0y + 16z = -16$ ⑧

⇒ $z = -1$

$\overset{r_2}{\Rightarrow}$ $y = 1$

Sub into ①

$x = 2$

∴ our sol'n is $(x, y, z) = (2, 1, -1)$

Class/Homework for Section 9.2

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