# Chapter 6 Review

**d.** true; Draw the parallelogram formed by $\vec{RF}$ and $\vec{SW}$. $\vec{FW}$ and $\vec{RS}$ are the opposite sides of a parallelogram and must be equal.

**e.** true; the distributive law for scalars

**f.** false; Let $\vec{b} = -\vec{a}$ and let $\vec{c} = \vec{d} \neq 0$. Then, $|\vec{a}| = |-\vec{a}| = |\vec{b}|$ and $|\vec{c}| = |\vec{d}|$ but $|\vec{a} + \vec{b}| = |\vec{a} + (-\vec{a})| = 0$

$|\vec{c} + \vec{b}| = |\vec{c} + \vec{c}| = |2\vec{c}|$

so $|\vec{a} + \vec{b}| \neq |\vec{c} + \vec{d}|$

**2.** **a.** $20\vec{a} - 30\vec{b} + 8\vec{c}$

**b.** $\vec{a} - 3\vec{b} - 3\vec{c}$

**3.** **a.** $\vec{XY} = (-2, 3, 6)$,

$|\vec{XY}| = 7$

**b.** $\left(-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

**4.** **a.** $(-6, -3, -6)$

**b.** $\left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

**5.** $\left(-\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

**6.** **a.** $\vec{OA} + \vec{OB} = (-3, 8, -8)$,

$\vec{OA} - \vec{OB} = (9, -4, -4)$

**b.** $\theta \doteq 84.4°$

**7.** **a.** $|\vec{AB}| = \sqrt{14}$,

$|\vec{BC}| = \sqrt{59}$,

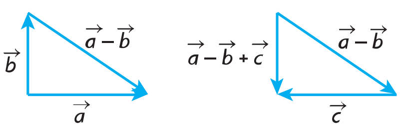
$|\vec{CA}| = \sqrt{45}$

**b.** 12.5

**c.** 18.13

**d.** $(6, 2, -2)$

**8.** **a.**




**b.** 5

**9.** $\frac{1}{2}(-11, 7) + \left(-\frac{3}{2}\right)(-3, 1) = (-1, 2)$,

$\frac{1}{3}(-11, 7) + \left(-\frac{2}{3}\right)(-1, 2) = (-3, 1)$,

$3(-3, 1) + 2(-1, 2) = (-11, 7)$

**10.** **a.** $x - 3y + 6z = 0$ where $P(x, y, z)$ is the point.

**b.** $(0, 0, 0)$ and $\left(1, \frac{1}{3}, 0\right)$

**11.** **a.** $a = -3, b = 26.5, c = 10$

**b.** $a = 8, b = \frac{7}{3}, c = -10$

**12.** **a.** yes

**b.** yes

**13.** **a.** $|\vec{AB}|^2 = 9, |\vec{AC}|^2 = 3, |\vec{BC}|^2 = 6$

Since $|\vec{AB}|^2 = |\vec{AC}|^2 + |\vec{BC}|^2$ the triangle is right-angled

**b.** $\frac{\sqrt{6}}{3}$

**14.** **a.** $\vec{DA}, \vec{BC}$ and $\vec{EB}, \vec{ED}$

**b.** $\vec{DC}, \vec{AB}$ and $\vec{CE}, \vec{EA}$

**c.** $|\vec{AD}|^2 + |\vec{DC}|^2 = |\vec{AC}|^2$

But $|\vec{AC}|^2 = |\vec{DB}|^2$

Therefore, $|\vec{AD}|^2 + |\vec{DC}|^2 = |\vec{DB}|^2$

**15.** **a.** $C(3, 0, 5)$, $P(3, 4, 5)$, $E(0, 4, 5)$, $F(0, 4, 0)$

**b.** $\vec{DB} = (3, 4, -5)$,

$\vec{CF} = (-3, 4, -5)$

**c.** 90°

**d.** 50.2°

**16.** **a.** 7.74

**b.** 2.83

**c.** 2.83

**17.** **a.** 1236.9 km

**b.** S14.0°W

**18.** **a.** Any pair of nonzero, noncollinear vectors will span $R^2$. To show that (2, 3) and (3, 5) are noncollinear, show that there does not exist any number $k$ such that $k(2, 3) = (3, 5)$. Solve the system of equations:

$2k = 3$

$3k = 5$

Solving both equations gives two different values for $k$, $\frac{3}{2}$ and $\frac{5}{3}$, so (2, 3) and (3, 5) are noncollinear and thus span $R^2$.

**b.** $m = -770, n = 621$

**19.** **a.** Find $a$ and $b$ such that

$$(5, 9, 14) = a(-2, 3, 1) + b(3, 1, 4)$$
$$(5, 9, 14) = (-2a, 3a, a) + (3b, b, 4b)$$
$$(5, 9, 14) = (-2a + 3b, 3a + b, a + 4b)$$

**i.** $5 = -2a + 3b$

**ii.** $9 = 3a + b$

**iii.** $14 = a + 4b$

Use the method of elimination with i. and iii.

$$\begin{aligned} 2(14) &= 2(a + 4b) \\ 28 &= 2a + 8b \\ +\quad 5 &= -2a + 3b \\ \hline 33 &= 11b \\ 3 &= b \end{aligned}$$

By substitution, $a = 2$.

$\vec{a}$ lies in the plane determined by $\vec{b}$ and $\vec{c}$ because it can be written as a linear combination of $\vec{b}$ and $\vec{c}$.

**b.** If vector $\vec{a}$ is in the span of $\vec{b}$ and $\vec{c}$, then $\vec{a}$ can be written as a linear combination of $\vec{b}$ and $\vec{c}$. Find $m$ and $n$ such that

$$\begin{aligned} (-13, 36, 23) &= m(-2, 3, 1) + n(3, 1, 4) \\ &= (-2m, 3m, m) + (3n, n, 4n) \\ &= (-2m + 3n, 3m + n, m + 4n) \end{aligned}$$

Solve the system of equations:

$-13 = -2m + 3n$

$36 = 3m + n$

$23 = m + 4n$

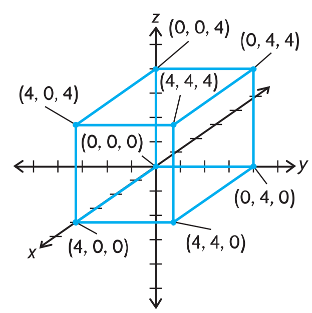
Use the method of elimination:

$$\begin{aligned} 2(23) &= 2(m + 4n) \\ 46 &= 2m + 8n \\ +\ -13 &= -2m + 3n \\ \hline 33 &= 11n \\ 3 &= n \end{aligned}$$

By substitution, $m = 11$.

So, vector $\vec{a}$ is in the span of $\vec{b}$ and $\vec{c}$.

**20.** **a.**




**b.** $(-4, -4, -4)$

**c.** $(-4, 0, -4)$

**d.** $(4, 4, 0)$

**21.** 7

**22.** **a.** $|\vec{AB}| = 10$,

$|\vec{BC}| = 2\sqrt{5} = 4.47$,

$|\vec{CA}| = \sqrt{80} = 8.94$

**b.** If $A$, $B$, and $C$ are vertices of a right triangle, then

$$|\vec{BC}|^2 + |\vec{CA}|^2 = |\vec{AB}|^2$$
$$\begin{aligned} |\vec{BC}|^2 + |\vec{CA}|^2 &= (2\sqrt{5})^2 + (\sqrt{80})^2 \\ &= 20 + 80 \\ &= 100 \\ |\vec{AB}|^2 &= 10^2 \\ &= 100 \end{aligned}$$

So, triangle $ABC$ is a right triangle.

**23.** **a.** $\vec{a} + \vec{b} + \vec{c}$

**b.** $\vec{a} - \vec{b}$

**c.** $-\vec{b} - \vec{a} + \vec{c}$

**d.** $\vec{0}$

**e.** $\vec{b} + \vec{c}$

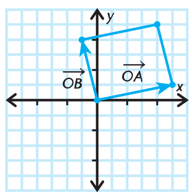
# Chapter 7 Review

**2.** **a.** 3
**b.** 7
**c.** $4\sqrt{3}$
**d.** $2\sqrt{17}$
**e.** 5
**f.** $-1$

**3.** **a.** 6
**b.** $-\frac{54}{5}$

**4.** about 18.52°

**5.** **a.**

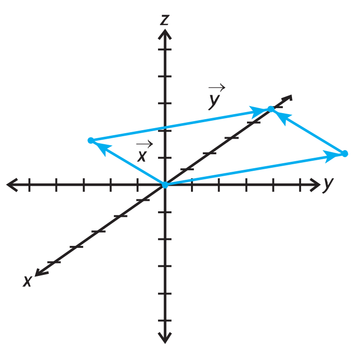



**b.** about 77.9°

**6.** rope at 45°: about 87.86 N,
rope at 30°: about 71.74 N

**7.** 304.14 km/h, W 9.46° N

**8.** **a.**



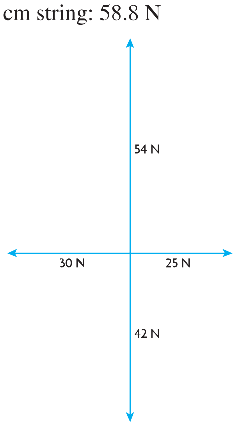

**b.** approximately 56.78

**9.** $\left(\frac{9}{\sqrt{115}}, -\frac{5}{\sqrt{115}}, -\frac{3}{\sqrt{115}}\right)$

**10.** **a.** about 77.64° is the largest angle
**b.** 36.50

**11.** 30 cm string: 78.4 N;
40 cm string: 58.8 N

**12.** **a.**




**b.** The resultant is 13 N in a direction N22.6°W. The equilibrant is 13 N in a direction S22.6°E.

**13.** **a.** Let $D$ be the origin, then:
$A = (2, 0, 0), B = (2, 4, 0),$
$C = (0, 4, 0), D = (0, 0, 0),$
$E = (2, 0, 3), F = (2, 4, 3),$
$G = (0, 4, 3), H = (0, 0, 3)$
**b.** about 44.31°
**c.** about 3.58

**14.** 7.5

**15.** **a.** about 48.2°
**b.** about 8 min 3 s
**c.** Such a situation would have resulted in a right triangle where one of the legs is longer than the hypotenuse, which is impossible.

**16.** **a.** $\overrightarrow{OA} + \overrightarrow{OB} = (-3, 8, -8),$
$\overrightarrow{OA} - \overrightarrow{OB} = (9, -4, -4)$
**b.** about 84.36°

**17.** **a.** $a = 4$ and $b = -4$
**b.** $\vec{p} \cdot \vec{q} = 2a - 2b - 18 = 0$
**c.** $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$

**18.** **a.** about 74.62°
**b.** about 0.75
**c.** $(0.1875)(\sqrt{3}, -2, -3)$
**d.** about 138.59°

**19.** **a.** special
**b.** not special

**20.** **a.** $(-1, 1, 3)$
**b.** $(-2, 2, 6)$
**c.** 0
**d.** $(-1, 1, 3)$

**21.** about 11.55 N

**22.** $(2, -8, -10)$

**23.** $-141$

**24.** 5 or $-7$

**25.** about 103.34°

**26.** **a.** $C = (3, 0, 5), F = (0, 4, 0)$
**b.** $(-3, 4, -5)$
**c.** about 111.1°

**27.** **a.** about 7.30
**b.** about 3.84
**c.** about 3.84

**28.** **a.** scalar: 1,
vector: $\vec{i}$
**b.** scalar: 1,
vector: $\vec{j}$
**c.** scalar: $\frac{1}{\sqrt{2}}$,
vector: $\frac{1}{2}(\vec{k} + \vec{j})$

**29.** **a.** $|\vec{b}|, |\vec{c}|$
**b.** $\vec{a}$; When dotted with $\vec{d}$, it equals 0.

**30.** 7.50 J

**31.** **a.** $\vec{a} \cdot \vec{b} = 6 - 5 - 1 = 0$
**b.** $\vec{a}$ with the $x$-axis:
$|\vec{a}| = \sqrt{4 + 25 + 1} = \sqrt{30}$
$\cos(\alpha) = \frac{2}{\sqrt{30}}$
$\vec{a}$ with the $y$-axis:
$\cos(\beta) = \frac{5}{\sqrt{30}}$
$\vec{a}$ with the $z$-axis:
$\cos(\gamma) = \frac{-1}{\sqrt{30}}$
$|\vec{b}| = \sqrt{9 + 1 + 1} = \sqrt{11}$
$\vec{b}$ with the $x$-axis:
$\cos(\alpha) = \frac{3}{\sqrt{11}}$
$\vec{b}$ with the $y$-axis:
$\cos(\beta) = \frac{-1}{\sqrt{11}}$
$\vec{b}$ with the $z$-axis:
$\cos(\gamma) = \frac{1}{\sqrt{11}}$
**c.** $\vec{m}_1 \times \vec{m}_2 = \frac{6}{\sqrt{330}} - \frac{5}{\sqrt{330}}$
$- \frac{1}{\sqrt{330}} = 0$

**32.** $|3\vec{i} + 3\vec{j} + 10\vec{k}| = \sqrt{118}$
$|-\vec{i} + 9\vec{j} - 6\vec{k}| = \sqrt{118}$

**33.** **a.** $\cos\alpha = \frac{\sqrt{3}}{2}$,
$\cos\beta = \cos\gamma = \pm\frac{1}{2\sqrt{2}}$
**b.** acute case: 69.3°,
obtuse case: 110.7°

**34.** $-5$

**35.** $|\vec{a} + \vec{b}| = \sqrt{1 + 1 + 64} = \sqrt{66}$
$|\vec{a} - \vec{b}| = \sqrt{1 + 81 + 16} = \sqrt{98}$
$\frac{1}{4}|\vec{a} + \vec{b}|^2 - \frac{1}{4}|\vec{a} - \vec{b}|^2$
$= \frac{66}{4} - \frac{98}{4} = -8$

**36.** $\vec{c} = \vec{b} - \vec{a}$
$|\vec{c}|^2 = |\vec{b} - \vec{a}|^2$
$= (\vec{b} - \vec{a})(\vec{b} - \vec{a})$
$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$
$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

**37.** $\overrightarrow{AB} = (2, 0, 4)$
$|\overrightarrow{AB}| = 2\sqrt{5}$
$\overrightarrow{AC} = (1, 0, 2)$
$|\overrightarrow{AC}| = \sqrt{5}$
$\overrightarrow{BC} = (-1, 0, -2)$
$|\overrightarrow{BC}| = \sqrt{5}$

CHAPTER 8 REVIEW

## Review Exercise, pp. 480–483

**1.** Answers may vary. For example,
$x = 1 + s + t,$
$y = 2 - s,$
$z = -1 + 2s + 3t$

**2.** $3x + y - z - 6 = 0$
$\overrightarrow{AC} = (2, -1, 5) = \vec{c}$
$\overrightarrow{BC} = (1, 0, 3) = \vec{b}$
$\vec{r} = (1, 2, -1) + s(2, -1, 5) + t(1, 0, 3),\ s, t \in \mathbf{R}$
$\vec{b} \times \vec{c} = (1, 0, 3) \times (2, -1, 5) = (3, 1, -1)$
$Ax + By + Cz + D = 0$
$(3)x + (1)y + (-1)z + D = 0$
$3(1) + (2) - 1(-1) + D = 0$
$D = -6$
$3x + y - z - 6 = 0$
Both Cartesian equations are the same regardless of which vectors are used.

**3. a.** Answers may vary. For example,
$\vec{r} = (4, 3, 9) + t(7, 1, 1),\ t \in \mathbf{R};$
$x = 4 + 7t,\ y = 3 + t,\ z = 9 + t,\ t \in \mathbf{R};$
$\frac{x - 4}{7} = \frac{y - 3}{1} = \frac{z - 9}{1}$

**b.** Answers may vary. For example,
$\vec{r} = (4, 3, 9) + t(7, 1, 1) + s(3, 2, 3),\ t, s \in \mathbf{R};$
$x = 4 + 7t + 3s,\ y = 3 + t + 2s,$
$z = 9 + t + 3s,\ t, s \in \mathbf{R}$

**c.** There are two parameters.

**4.** $\vec{r} = (7, 1, -2) + t(2, -3, 1),\ t \in \mathbf{R};$
$x = 7 + 2t,\ y = 1 - 3t,\ z = -2 + t;$
$\frac{x - 7}{2} = \frac{y - 1}{-3} = \frac{z + 2}{1}$

**5. a.** $x - 3y - 3z - 3 = 0$
**b.** $3x + 5y - 2z - 7 = 0$
**c.** $3y + z - 7 = 0$

**6.** $19x - 7y - 8z = 0$

**7.** $\vec{r} = (-1, 2, 1) + t(0, 1, 0) + s(0, 0, 1)\ t, s \in \mathbf{R};$
$x = -1,\ y = 2 + t,\ z = 1 + s$

**8.** $3x + y - z - 7 = 0$

**9.** $34x + 32y - 7z - 229 = 0$

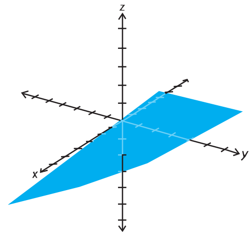
**10.** Answers may vary. For example,
$\vec{r} = (2, 3, -3) + s(3, -2, 1),\ s \in \mathbf{R};$
$x = 2 + 3s,\ y = 3 - 2s,\ z = -3 + s;$
$\frac{x - 2}{3} = \frac{y - 3}{-2} = \frac{z + 3}{1}$

**11.** Answers may vary. For example,
$\vec{r} = (0, 0, 6) + s(1, 0, 3) + t(3, -5, -1),\ s, t \in \mathbf{R};$
$x = s + 3t,\ y = -5t,\ z = 6 + 3s - t$

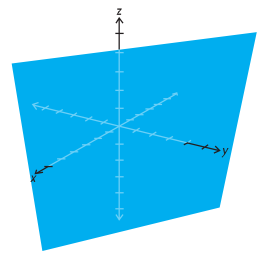
**12.** Answers may vary. For example,
$\vec{r} = (0, 0, 7) + t(1, 0, 2),\ t \in \mathbf{R};$
$x = t,\ y = 0,\ z = 7 + 2t;$

**13.** $\vec{r} = (3, -4, 1) + s(1, -3, -5) + t(4, 3, -1),\ s, t \in \mathbf{R};$
$x = 3 + s + 4t,$
$y = -4 - 3s + 3t$
$z = 1 - 5s - t,\ s, t \in \mathbf{R};$
$18x - 19y + 15z - 145 = 0$

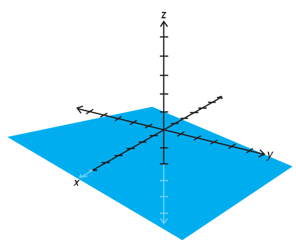
**14. a.**



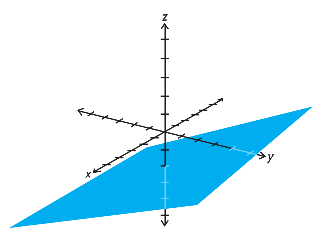
**b.**



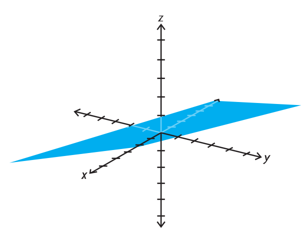
**c.**



**d.**



**e.**



**15. a.** Answers may vary. For example,
$\vec{r} = (3, 1, 2) + t(2, 4, 1) + s(2, 3, -3),\ t, s \in \mathbf{R};$
$x = 3 + 2t + 2s,$
$y = 1 + 4t + 3s,\ z = 2 + t - 3s;$
$15x - 8y + 2z - 41 = 0$

**b.** Answers may vary. For example,
$\overrightarrow{BC} = (-4, 0, 11)$
$D = -18$
$-4x + 11z - 18 = 0$

**c.** Answers may vary. For example,
$\vec{r} = (4, 1, -1) + t(1, -3, 5) + s(0, 0, 1),\ t, s \in \mathbf{R};$
$x = 4 + t,\ y = 1 - 3t,$
$z = -1 + 5t + s;$
$3x + y - 13 = 0$

**d.** Answers may vary. For example,
$\vec{r} = (1, 3, -5) + t(1, 3, 9) + s(1, -9, -1),\ t, s \in \mathbf{R};$
$x = 1 + t + s,\ y = 3 + 3t - 9s,$
$z = -5 + 9t - s;$
$78x + 10y - 12z - 168 = 0$

**16.** They are in the same plane because both planes have the same normal vectors and Cartesian equations.
$L_1$: $\vec{r} = (1, 2, 3) + s(-3, 5, 21) + t(0, 1, 3),\ s, t \in \mathbf{R}$
$L_2$: $\vec{r} = (1, -1, -6) + u(1, 1, 1) + v(2, 5, 11),\ u, v \in \mathbf{R}$
$(-3, 5, 21) \times (0, 1, 3) = (-6, 9, -3) = (2, -3, 1)$
$(1, 1, 1) \times (2, 5, 11) = (6, -9, 3) = (2, -3, 1)$
$Ax + By + Cz + D = 0$
$2x - 3y + z + D = 0$
$2(1) - 3(2) + (3) + D = 0$
$D = 1$
$2(1) - 3(-1) + (-6) + D = 0$
$D = 1$
$2x - 3y + z + 1 = 0$

**17.** $\left(\frac{20}{3}, \frac{10}{3}, -\frac{1}{3}\right)$

**18. a.** The plane is parallel to the $z$-axis through the points $(3, 0, 0)$ and $(0, -2, 0)$.
**b.** The plane is parallel to the $y$-axis through the points $(6, 0, 0)$ and $(0, 0, -2)$.
**c.** The plane is parallel to the $x$-axis through the points $(0, 3, 0)$ and $(0, 0, -6)$.

**19. a.** $A$
**b.** $a = -8,\ b = -1$

**20. a.** 45.0° **c.** 37.4°
**b.** 59.0° **d.** 90°

**21. a.** 44.2°
**b.** 90°

**22. a. i.** no **ii.** yes **iii.** no
**b. i.** yes **ii.** no **iii.** no

**23.** $(x, y, z) = (4, 1, 6) + p(3, -2, 1) + q(-6, 6, -1)$
$(x, y, z) = (4, 1, 6) + 4(3, -2, 1) + 2(-6, 6, -1)$
$(x, y, z) = (4, 5, 8) \neq (4, 5, 6)$

**24.** $x = 1 + s + 3t,\ y = 4 - t,$
$z = 4 - 3s + t,\ s, t \in \mathbf{R}$

25. A plane has two parameters, because a plane goes in two different directions, unlike a line that goes only in one direction.

26. This equation will always pass through the origin, because you can always set $s = 0$ and $t = -1$ to obtain $(0, 0, 0)$.

27. **a.** They do not form a plane, because these three points are collinear. $\vec{r} = (-1, 2, 1) + t(3, 1, -2)$

**b.** They do not form a plane, because the point lies on the line.
$\vec{r} = (4, 9, -3) + t(1, -4, 2)$
$\vec{r} = (4, 9, -3) + 4(1, -4, 2) = (8, -7, 5)$

28. $bcx + acy + abz - abc = 0$

29. $6x - 5y + 12z + 46 = 0$

30. **a., b.** $\vec{r} = (1, -3, 2) + t(-3, 7, -4) + s(5, -2, 3)$ $t, s \in \mathbf{R}$;
$x = 1 - 3t + 5s$,
$y = -3 + 7t - 2s$,
$z = 2 - 4t + 3s$

**c.** $13x - 11y - 29z + 12 = 0$

**d.** no

31. **a.** $4x - 2y + 5z = 0$

**b.** $4x - 2y + 5z + 19 = 0$

**c.** $4x - 2y + 5z - 22 = 0$

32. **a.** These lines are coincident. The angle between them is 0°.

**b.** $\left(\frac{3}{2}, 5\right)$, 86.82°

33. **a.** $\vec{r} = (1, 3, 5) + t(-2, -4, -10)$, $t \in \mathbf{R}$;
$x = 1 - 2t, y = 3 - 4t$,
$z = 5 - 10t$;
$\frac{x-1}{-2} = \frac{y-3}{-4} = \frac{z-5}{-10}$

**b.** $\vec{r} = (1, 3, 5) + t(-8, 6, -2)$, $t \in \mathbf{R}$;
$x = 1 - 8t, y = 3 + 6t$,
$z = 5 - 2t$;
$\frac{x-1}{-8} = \frac{x-3}{6} = \frac{x-5}{-2}$

**c.** $\vec{r} = (1, 3, 5) + t(-6, -13, 14)$, $t \in \mathbf{R}$;
$x = 1 - 6t, y = 3 - 13t$,
$z = 5 + 14t$;
$\frac{x-1}{-6} = \frac{x-3}{-13} = \frac{x-5}{14}$

**d.** $\vec{r} = (1, 3, 5) + t(1, 0, 0)$, $t \in \mathbf{R}$;
$x = 1 + t, y = 3, z = 5$

**e.** $a = 0, b = 6, c = 4$;
$\vec{r} = (1, 3, 5) + t(0, 6, 4)$, $t \in \mathbf{R}$

**f.** $\vec{r} = (1, 3, 5) + t(0, 1, 6)$;
$x = 1, y = 3 + t, z = 5 + 6t$

34. **a.** $2x - 4y + 5z + 23 = 0$

**b.** $29x + 27y + 24z - 86 = 0$

**c.** $z - 3 = 0$

**d.** $3x + y - 4z + 26 = 0$

**e.** $y - 2z - 4 = 0$

**f.** $-5x + y + 7z + 18 = 0$

## Chapter 8 Test, p. 484

1. **a. i.** $\vec{r} = (1, 2, 4) + s(1, -2, -1) + t(3, 2, 0)$, $s, t \in \mathbf{R}$;
$x = 1 + s + 3t$,
$y = 2 - 2s + 2t, z = 4 - s$,
$s, t \in \mathbf{R}$

**ii.** $2x - 3y + 8z - 28 = 0$

**b.** no

2. **a.** $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

**b.** $(6, 4, 3)$

3. **a.** $\vec{r} = s(2, 1, 3) + t(1, 2, 5)$, $s, t \in \mathbf{R}$

**b.** $-x - 7y + 3z = 0$

4. **a.** $\vec{r} = (4, -3, 5) + s(2, 0, -3) + t(5, 1, -1)$, $s, t \in \mathbf{R}$

**b.** $3x - 13y + 2z - 61 = 0$

5. **a.** $\left(0, 5, -\frac{1}{2}\right)$

**b.** $\frac{x}{4} = \frac{y-5}{-2} = \frac{1}{2}$

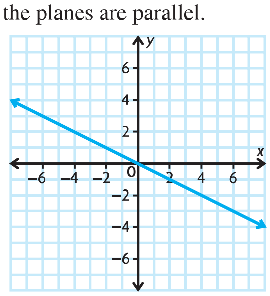
6. **a.** about 70.5°

**b. i.** 4

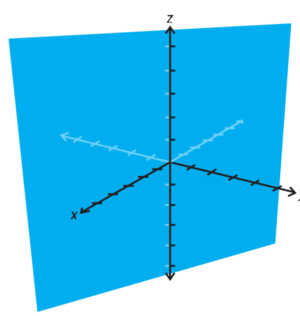
**ii.** $-\frac{1}{5}$

**c.** The $y$-intercepts are different and the planes are parallel.

7. **a.**




**b.**




**c.** The equation for the plane can be written as $Ax + By + 0z = 0$. For any real number $t$, $A(0) + B(0) + 0(t) = 0$, so $(0, 0, t)$ is on the plane. Since this is true for all real numbers, the $z$-axis is on the plane.

# Chapter 9

## Review of Prerequisite Skills, p. 487

1. **a.** yes **b.** no **c.** yes **d.** no

2. Answers may vary. For example:

**a.** $\vec{r} = (2, 5) + t(5, -2)$, $t \in \mathbf{R}$;
$x = 2 + 5t, y = 5 - 2t$, $t \in \mathbf{R}$

**b.** $\vec{r} = (-3, 7) + t(7, -14)$, $t \in \mathbf{R}$;
$x = -3 + 7t, y = 7 - 14t$, $t \in \mathbf{R}$

**c.** $\vec{r} = (-1, 0) + t(-2, -11)$, $t \in \mathbf{R}$;
$x = -1 + -2t, y = -11t$, $t \in \mathbf{R}$

**d.** $\vec{r} = (1, 3, 5) + t(5, -10, -5)$, $t \in \mathbf{R}$;
$x = 1 + 5t, y = 3 - 10t, z = 5 - 5t$, $t \in \mathbf{R}$

**e.** $\vec{r} = (2, 0, -1) + t(-3, 5, 3)$, $t \in \mathbf{R}$;
$x = 2 - 3t, y = 5t, z = -1 + 3t$, $t \in \mathbf{R}$

**f.** $\vec{r} = (2, 5, -1) + t(10, -10, -6)$, $t \in \mathbf{R}$;
$x = 2 + 10t, y = 5 - 10t, z = -1 - 6t$, $t \in \mathbf{R}$

3. **a.** $2x + 6y - z - 17 = 0$

**b.** $y = 0$

**c.** $4x - 3y - 15 = 0$

**d.** $6x - 5y + 3z = 0$

**e.** $11x - 6y - 38 = 0$

**f.** $x + y - z - 6 = 0$

4. $5x + 11y + 2z - 21 = 0$

5. $L_1$ is not parallel to the plane. $L_1$ is on the plane.
$L_2$ is parallel to the plane.
$L_3$ is not parallel to the plane.

6. **a.** $x - y - z - 2 = 0$

**b.** $x + 6y - 10z - 30 = 0$

7. $\vec{r} = (1, -4, 3) + t(1, 3, 3) + s(0, 1, 0)$, $s, t \in \mathbf{R}$

8. $3y + z = 13$

## Section 9.1, pp. 496–498

1. **a.** $\pi: x - 2y - 3z = 6$,
$\vec{r} = (1, 2, -3) + s(5, 1, 1)$ $s \in \mathbf{R}$

**b.** This line lies on the plane.

2. **a.** A line and a plane can intersect in three ways: (1) The line and the plane have zero points of intersection. This occurs when the lines are not incidental, meaning they do not intersect.
(2) The line and the plane have only one point of intersection. This occurs when the line crosses the plane at a single point.
(3) The line and the plane have an infinite number of intersections. This occurs when the line is

CHAPTER 9 REVIEW AND CUMULATIVE REVIEW

6. $\vec{r} = (3,1,1) + t(2,-1,2),\ t \in \mathbf{R}$

8. a. $x = -\frac{5}{7}t,\ y = 1 + \frac{2}{7}t,\ z = t,\ t \in \mathbf{R}$
   b. $x = 3,\ y = \frac{1}{4},\ z = -\frac{1}{2}$
   c. $x = 3t - 3s + 7,\ y = t,\ z = s,$ $s, t \in \mathbf{R}$

9. a. $x = \frac{1}{2} + \frac{1}{36}t,\ y = -\frac{1}{2} + \frac{5}{12}t,$ $z = t,\ t \in \mathbf{R}$
   b. $x = \frac{9}{8} - \frac{31}{24}t,\ y = \frac{1}{4} + \frac{1}{12}t,\ z = t,$ $t \in \mathbf{R}$

10. a. These three planes meet at the point $(-1, 5, 3)$.
    b. The planes do not intersect. Geometrically, the planes form a triangular prism.
    c. The planes meet in a line through the origin, with equation $x = t,$ $y = -7t,\ z = -5t,\ t \in \mathbf{R}$

11. 4.90

12. a. $x - 2y + z + 4 = 0$
    $\vec{r} = (3, 1, -5) + s(2, 1, 0),\ s \in \mathbf{R}$
    $\vec{m} \times \vec{n} = (2, 1, 0)(1, -2, 1) = 0$
    Since the line's direction vector is perpendicular to the normal of the plane and the point $(3, 1, -5)$ lies on both the line and the plane, the line is in the plane.
    b. $(-1, -1, -5)$
    c. $x - 2y + z + 4 = 0$
    $-1 - 2(-1) + (-5) + 4 = 0$
    The point $(-1, -1, -5)$ is on the plane since it satisfies the equation of the plane.
    d. $7x - 2y - 11z - 50 = 0$

13. a. 5.48
    b. $(3, 0, -1)$

14. a. $(-2, -3, 0)$.
    b. $\vec{r} = (-2, -3, 0) + t(1, -2, 1),$ $t \in \mathbf{R}$

15. a. $-10x + 9y + 8z + 16 = 0$
    b. about 0.45

16. a. 1
    b. $\vec{r} = (0, 0, -1) + t(4, 3, 7),\ t \in \mathbf{R}$

17. a. $x = 2,\ y = -1,\ z = 1$
    b. $x = 7 - 3t,\ y = 3 - t,\ z = t,\ t \in \mathbf{R}$

18. $a = \frac{2}{3},\ b = \frac{3}{4},\ c = \frac{1}{2}$

19. $\left(4, -\frac{7}{4}, \frac{7}{2}\right)$

20. $\left(-\frac{5}{3}, \frac{8}{3}, \frac{4}{3}\right)$

21. a. $\vec{r} = \left(\frac{45}{4}, 0, -\frac{21}{4}\right) + t(11, 2, -5),\ t \in \mathbf{R};$
    $\vec{r} = \left(-\frac{37}{2}, 0, \frac{15}{2}\right) + t(11, 2, -5),\ t \in \mathbf{R};$
    $\vec{r} = (7, 0, -1) + t(11, 2, -5),$ $t \in \mathbf{R};\ z = -1 - 5t,\ t \in \mathbf{R}$
    b. All three lines of intersection found in part a. have direction vector $(11, 2, -5)$, and so they are all parallel. Since no pair of normal vectors for these three planes is parallel, no pair of these planes is coincident.

22. $\left(\frac{1}{2}, 1, \frac{1}{3}\right), \left(\frac{1}{2}, 1, -\frac{1}{3}\right), \left(\frac{1}{2}, -1, \frac{1}{3}\right),$ $\left(\frac{1}{2}, -1, -\frac{1}{3}\right), \left(-\frac{1}{2}, 1, \frac{1}{3}\right),$ $\left(\frac{1}{2}, -1, -\frac{1}{3}\right)\ \left(-\frac{1}{2}, 1, -\frac{1}{3}\right)$, and $\left(-\frac{1}{2}, -1, \frac{1}{3}\right)$

23. $y = \frac{7}{6}x^2 - \frac{3}{2}x - \frac{2}{3}$

24. $\left(\frac{29}{7}, \frac{4}{7}, -\frac{33}{7}\right)$

25. $A = 5, B = 2, C = -4$

26. a. $\vec{r} = (-1, -4, -6) + t(-5, -4, -3),\ t \in \mathbf{R}$
    b. $\left(\frac{13}{2}, 2, -\frac{3}{2}\right)$
    c. about 33.26 units$^2$

27. $6x - 8y + 9z - 115 = 0$

## Chapter 9 Test, p. 556

1. a. $(3, -1, -5)$
   b. $3 - (-1) + (-5) + 1 = 0$
   $3 + 1 - 5 + 1 = 0$
   $0 = 0$

2. a. $\frac{13}{12}$ or 1.08
   b. $\frac{40}{3}$ or 13.33

3. a. $x = \frac{4t}{5},\ y = 1 - \frac{t}{5},\ z = t,\ t \in \mathbf{R}$
   b. $(4, 0, 5)$

4. a. $(1, -5, 4)$
   b. The three planes intersect at the point $(1, -5, 4)$.

5. a. $x = -\frac{1}{2} - \frac{t}{4},\ y = \frac{3t}{4} + \frac{1}{2},\ z = t,$ $t \in \mathbf{R}$
   b. The three planes intersect at this line.

6. a. $m = -1, n = -3$
   b. $x = -1,\ y = 1 - t,\ z = t,\ t \in \mathbf{R}$

7. 10.20

## Cumulative Review of Vectors, pp. 557–560

1. a. about 111.0°
   b. scalar projection: $-\frac{14}{13}$,
   vector projection: $\left(-\frac{52}{169}, \frac{56}{169}, -\frac{168}{169}\right)$
   c. scalar projection: $-\frac{14}{3}$,
   vector projection: $\left(-\frac{28}{9}, \frac{14}{9}, \frac{28}{9}\right)$

2. a. $x = 8 + 4t,\ y = t,\ z = -3 - 3t,$ $t \in \mathbf{R}$
   b. about 51.9°

3. a. $\frac{1}{2}$
   b. 3
   c. $\frac{3}{2}$

4. a. $-7\vec{i} - 19\vec{j} - 14\vec{k}$
   b. 18

5. $x$-axis: about 42.0°, $y$-axis: about 111.8°, $z$-axis: about 123.9°

6. a. $(-7, -5, -1)$
   b. $(-42, -30, -6)$
   c. about 8.66 square units
   d. 0

7. $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

8. a. vector equation: Answers may vary.
   $\vec{r} = (2, -3, 1) + t(-1, 5, 2),\ t \in \mathbf{R};$
   parametric equation:
   $x = 2 - t,\ y = -3 + 5t,$
   $z = 1 + 2t,\ t \in \mathbf{R}$
   b. If the $x$-coordinate of a point on the line is 4, then $2 - t = 4$, or $t = -2$. At $t = -2$, the point on the line is $(2, -3, 1) - 2(-1, 5, 2) = (4, -13, -3)$. Hence, $C(4, -13, -3)$ is a point on the line.

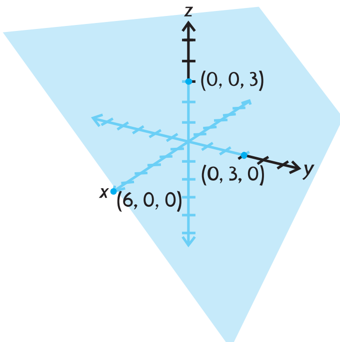
9. The direction vector of the first line is $(-1, 5, 2)$ and of the second line is $(1, -5, -2) = -(-1, 5, 2)$. So they are collinear and hence parallel. The lines coincide if and only if for any point on the first line and second line, the vector connecting the two points is a multiple of the direction vector for the lines. $(2, 0, 9)$ is a point on the first line and $(3, -5, 10)$ is a point on the second line. $(2, 0, 9) - (3, -5, 10) = (-1, 5, -1)$ $\neq k(-1, 5, 2)$ for $k \in \mathbf{R}$. Hence, the lines are parallel and distinct.

**10.** vector equation:
$\vec{r} = (0, 0, 4) + t(0, 1, 1), t \in \mathbf{R}$;
parametric equation: $x = 0, y = t,$
$z = 4 + t, t \in \mathbf{R}$

**11.** $-13$

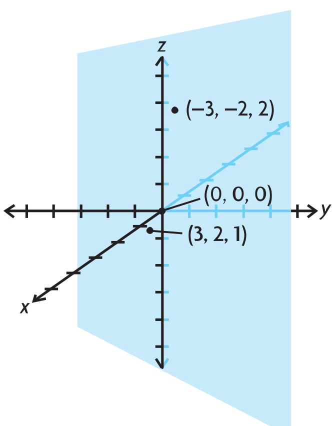
**12.** $\left(\frac{3}{2}, -\frac{31}{6}, \frac{13}{6}\right)$

**13.** **a.**



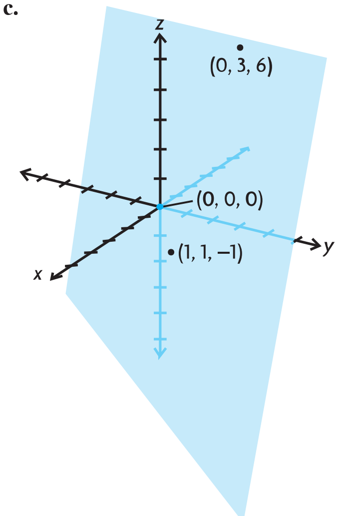

Answers may vary. For example, $(0, 3, -3)$ and $(6, 0, -3)$.

**b.**




Answers may vary. For example, $(-3, -2, 2)$ and $(3, 2, 1)$.

**c.**




Answers may vary. For example, $(0, 3, 6)$ and $(1, 1, -1)$.

**14.** $(-7, 10, 20)$

**15.** $\vec{q} = (1, 0, 2) + t(-11, 7, 2), t \in \mathbf{R}$

**16.** **a.** $12x - 9y - 6z + 24 = 0$
**b.** about 1.49 units

**17.** **a.** $3x - 5y + 4z - 7 = 0$
**b.** $x - y + 12z - 27 = 0$
**c.** $z - 3 = 0$
**d.** $x + 2z + 1 = 0$

**18.** 336.80 km/h, N 12.1° W

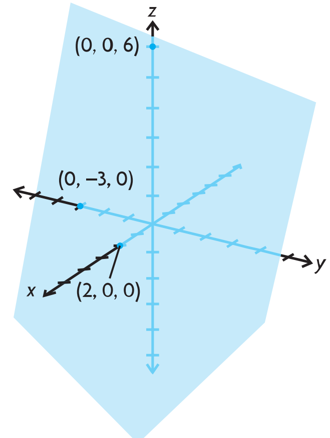
**19.** **a.** $\vec{r} = (0, 0, 6) + s(1, 0, -3) + t(0, 1, 2), s, t \in \mathbf{R}$. To verify, find the Cartesian equation corresponding to the above vector equation and see if it is equivalent to the Cartesian equation given in the problem. A normal vector to this plane is the cross product of the two directional vectors.

$\vec{n} = (1, 0, -3) \times (0, 1, 2)$

$= (0(2) - (-3)(1), -3(0) - 1(2), 1(1) - 0(0))$
$= (3, -2, 1)$

So the plane has the form $3x + 2y + z + D = 0$, for some constant $D$. To find $D$, we know that $(0, 0, 6)$ is a point on the plane, so $3(0) - 2(0) + (6) + D = 0$. So, $6 + D = 0$, or $D = -6$. So, the Cartesian equation for the plane is $3x - 2y + z - 6 = 0$. Since this is the same as the initial Cartesian equation, the vector equation for the plane is correct.

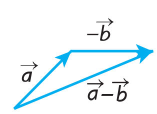
**b.**



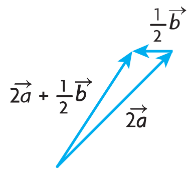

**20.** **a.** 16°
**b.** The two planes are perpendicular if and only if their normal vectors are also perpendicular. A normal vector for the first plane is $(2, -3, 1)$ and a normal vector for the second plane is $(4, -3, -17)$. The two vectors are perpendicular if and only if their dot product is zero.
$(2, -3, 1) \cdot (4, -3, -17)$
$= 2(4) - 3(-3) + 1(-17)$
$= 0$
Hence, the normal vectors are perpendicular. Thus, the planes are perpendicular.
**c.** The two planes are parallel if and only if their normal vectors are also parallel. A normal vector for the first plane is $(2, -3, 2)$ and a normal vector for the second plane is $(2, -3, 2)$. Since both normal vectors are the same, the planes are parallel. Since $2(0) - 3(-1) + 2(0) - 3 = 0$, the point $(0, -1, 0)$ is on the second plane. Yet since $2(0) - 3(-1) + 2(0) - 1 = 2 \neq 0$, $(0, -1, 0)$ is not on the first plane. Thus, the two planes are parallel but not coincident.

**21.** resultant: about 56.79 N, 37.6° from the 25 N force toward the 40 N force, equilibrant: about 56.79 N, 142.4° from the 25 N force away from the 40 N force

**22.** **a.**



**b.**




**23.** **a.** $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

**b.** $\left(-\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right)$

**24.** **a.** $\vec{OC} = (8, 9)$, $\vec{BD} = (10, -5)$

**b.** about 74.9°

**c.** about 85.6°

**25.** **a.** $x = t, y = -1 + t, z = 1, t \in \mathbf{R}$

**b.** $(1, 2, -3)$

**c.** $x = 1, y = t, z = -3 + t, t \in \mathbf{R}$

**d.** $x = 1 + 3s + t, y = t, z = s,$ $s, t \in \mathbf{R}$

**26.** **a.** yes; $x = 0, y = -1 + t, z = t, t \in \mathbf{R}$

**b.** no

**c.** yes; $x = 2 - 2t, y = t, z = 3t, t \in \mathbf{R}$

**27.** 30°

**28.** **a.** $-\frac{3}{2}$

**b.** 84

**29.** $\vec{r} = t(-1, 3, 1), t \in \mathbf{R}$, $-x + 3y + z - 11 = 0$

**30.** $(-1, 1, 0)$

**31.** **a.** 0.8 km

**b.** 12 min

**32.** **a.** Answers may vary. $\vec{r} = (6, 3, 4) + t(4, 4, 1), t \in \mathbf{R}$

**b.** The line found in part a will lie in the plane $x - 2y + 4z - 16 = 0$ if and only if both points $A(2, -1, 3)$ and $B(6, 3, 4)$ lie in this plane. We verify this by substituting these points into the equation of the plane, and checking for consistency.
For $A$:
$2 - 2(-1) + 4(3) - 16 = 0$
For $B$:
$6 - 2(3) + 4(4) - 16 = 0$
Since both points lie on the plane, so does the line found in part a.

**33.** 20 km/h at N 53.1° E

**34.** parallel: 1960 N, perpendicular: about 3394.82 N

**35.** **a.** True; all non-parallel pairs of lines intersect in exactly one point in $\mathbf{R}^2$. However, this is not the case for lines in $\mathbf{R}^3$ (skew lines provide a counterexample).

**b.** True; all non-parallel pairs of planes intersect in a line in $\mathbf{R}^3$.

**c.** True; the line $x = y = z$ has direction vector $(1, 1, 1)$, which is not perpendicular to the normal vector $(1, -2, 2)$ to the plane $x - 2y + 2z = k$, $k$ is any constant. Since these vectors are not perpendicular, the line is not parallel to the plane, and so they will intersect in exactly one point.

**d.** False; a direction vector for the line $\frac{x}{2} = y - 1 = \frac{z+1}{2}$ is $(2, 1, 2)$. A direction vector for the line $\frac{x-1}{-4} = \frac{y-1}{-2} = \frac{z+1}{-2}$ is $(-4, -2, -2)$, or $(2, 1, 1)$ (which is parallel to $(-4, -2, -2)$). Since $(2, 1, 2)$ and $(2, 1, 1)$ are obviously not parallel, these two lines are not parallel.

**36.** **a.** A direction vector for $L_1$: $x = 2, \frac{y-2}{3} = z$ is $(0, 3, 1)$, and a direction vector for $L_2$: $x = y + k = \frac{z+14}{k}$ is $(1, 1, k)$. But $(0, 3, 1)$ is not a nonzero scalar multiple of $(1, 1, k)$ for any $k$, since the first component of $(0, 3, 1)$ is 0. This means that the direction vectors for $L_1$ and $L_2$ are never parallel, which means that these lines are never parallel for any $k$.

**b.** 6; $(2, -4, -2)$

# Calculus Appendix

## Implicit Differentiation, p. 564

**1.** The chain rule states that if $y$ is a composite function, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$. To differentiate an equation implicitly, first differentiate both sides of the equation with respect to $x$, using the chain rule for terms involving $y$, then solve for $\frac{dy}{dx}$.

**2.** **a.** $-\frac{x}{y}$

**b.** $\frac{x^2}{5y}$

**c.** $\frac{-y^2}{2xy + y^2}$

**d.** $\frac{9x}{16y}$

**e.** $-\frac{13x}{48y}$

**f.** $-\frac{2x}{2y + 5}$

**3.** **a.** $y = \frac{2}{3}x - \frac{13}{3}$

**b.** $y = \frac{2}{3}(x + 8) + 3$

**c.** $y = -\frac{3\sqrt{3}}{5}x - 3$

**d.** $y = \frac{11}{10}(x + 11) - 4$

**4.** $(0, 1)$

**5.** **a.** 1

**b.** $\left(\frac{3}{\sqrt{5}}, \sqrt{5}\right)$ and $\left(-\frac{3}{\sqrt{5}}, -\sqrt{5}\right)$

**6.** $-10$

**7.** $7x - y - 11 = 0$

**8.** $y = \frac{1}{2}x - \frac{3}{2}$

**9.** **a.** $\frac{4}{(x + y)^2} - 1$

**b.** $4\sqrt{x + y} - 1$

**10.** **a.** $\frac{3x^2 - 8xy}{4x^2 - 3}$

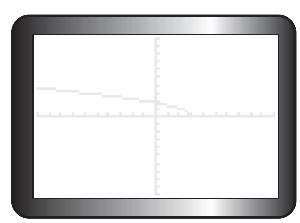
**b.** $y = \frac{x^3}{4x^2 - 3}$; $\frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$

**c.** $\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$

$y = \frac{x^3}{4x^2 - 3}$

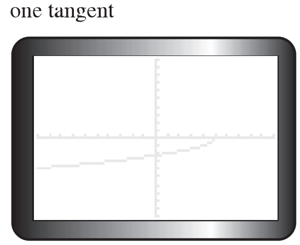
$$\frac{dy}{dx} = \frac{3x^2 - 8x\left(\frac{x^3}{4x^2 - 3}\right)}{4x^2 - 3}$$
$$= \frac{3x^2 - (4x^2 - 3) - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

**11.** **a.**



one tangent

**b.**



one tangent