Advanced Functions

Chapter 1: Introduction to Functions

Fall 2013

Course Notes

Chapter 1 – Introduction to Functions

Contents with suggested problems from the Nelson Textbook

1.1 Functions

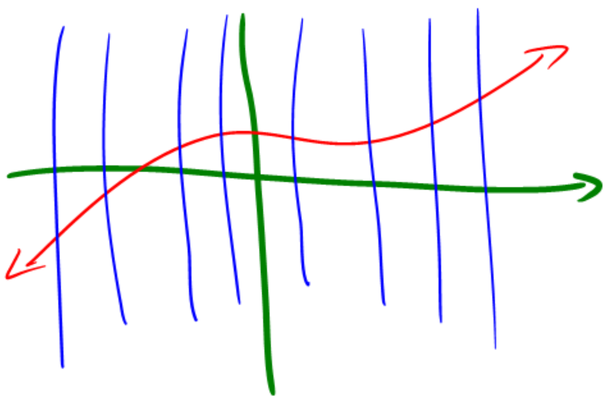
There are some people who argue that mathematics has just two basic building blocks: Numbers and Operations. This course is concerned with functions which can be considered number generators. A function takes a given number, and using mathematical operations generates another number. We will be examining the **relationship** between the given numbers, and the generated numbers for various functions.

Definition 1.1.1

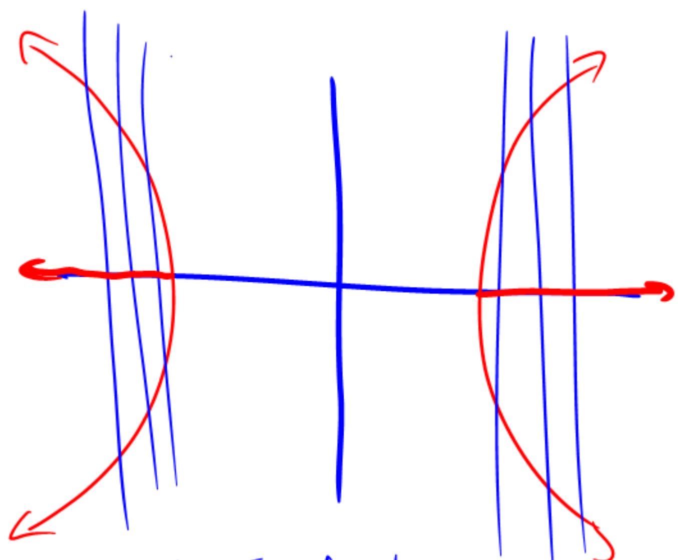
A **Function** is an algebraic rule which assigns exactly one m ber in a set called the range to each member of a set called the domain

Pictures

Vertical Line Test

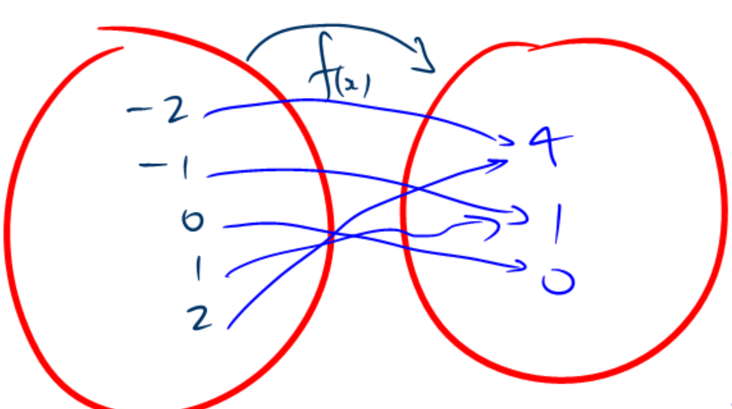


VLT passes



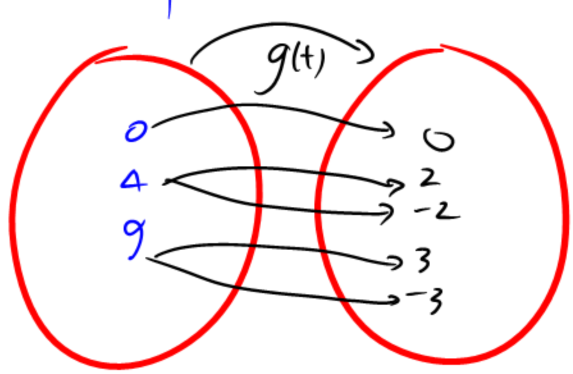
VLT fails

Arrow Diagrams



D_f R_f

fn!



D_g R_g

not a fn!

eg $f(x) = \frac{3}{x-2}$

Definition 1.1.2

Domain of a Function: the set of all values which "make sense" when "plugged into" the fn. $x \neq 2$

Range of a Function unless specified

the set of all (calculated) values depending on the fn rule and the specific domain values)

Function Notation

We use the notation $f(x)$ to "name" a function. This notation is powerful because it contains both the domain and the range. For example we might write $f(2)$, which shows that the domain value is $x = 2$, and that the range value (which we must calculate) is denoted $f(2)$.

Definition 1.1.3

The **Graph** of a function $f(x)$ is the set of all ordered pairs

$\{(x, f(x)) \mid x \in D_f\}$

Example 1.1.1

Given the graph of the function $f(x) = \{(3,4),(2,-1),(7,8),(4,2),(5,4)\}$ determine:

a) $D_f = \{3, 2, 7, 4, 5\}$

b) $R_f = \{4, -1, 8, 2\}$

c) Is $f(x)$ a function? Yes. The Defn is satisfied.

Example 1.1.2

Consider the sketch of the graph of $g(x)$, and determine:

a) $D_g = \{x \in \mathbb{R} \mid -4 \leq x \leq 5\}$

b) $R_g = \{g(x) \in \mathbb{R} \mid -2 \leq g(x) \leq 3\}$

c) Is $g(x)$ a function?

No. Fails VLT

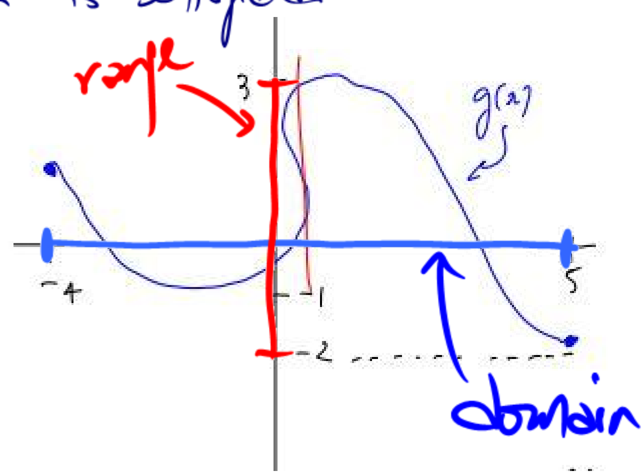


Figure 1.1.2

Note: In the above examples we have seen functions (and non-functions which we call relations) depicted graphically and numerically. We now turn to algebraic representations of functions. It is much more difficult to determine domain and range for functions given in an algebraic form, but the algebraic form is incredibly useful!

Example 1.1.3

State the domain and range of the functions given in algebraic form.

a) $f(x) = 3\cos(2x)$

b) $g(t) = (t-2)^2 + 1$ ⇒ vertex $(2,1)$ ← min value!

c) $h(x) = \dfrac{2}{x-1}$

a) $D_f: (-\infty, \infty)$ or $\{x \in \mathbb{R}\}$ or $x \in \mathbb{R}$

$R_f: [-3, 3]$ or $\{f(x) \in \mathbb{R} \mid -3 \le f(x) \le 3\}$ or $-3 \le f(x) \le 3$

b) $D_g: (-\infty, \infty)$ or $\{x \in \mathbb{R}\}$ or $x \in \mathbb{R}$

$R_g: [1, \infty)$ or $\{g(x) \in \mathbb{R} \mid g(x) \ge 1\}$ or $g(x) \ge 1$

c) $D_h: (-\infty, 1) \cup (1, \infty)$ ← union or $\{x \in \mathbb{R} \mid x \ne 1\}$ or $x \ne 1$

$R_h: (-\infty, 0) \cup (0, \infty)$ or ...

Notations for Domain and Range

use $g(x)$ above

Interval Notation

$D_g: [-4, 5]$ ← including end point

$R_g: [-2, 3]$

Set Notation

$D_g = \{x \in \mathbb{R} \mid -4 \le x \le 5\}$

Pseudo-set Notation

$D_g: -4 \le x \le 5$

Class/Homework for Section 1.1

Read Example 3 on Page 9
Pg. 11 – 13 #1 – 3, 5, 6, 7b-f, 9, 10, 12