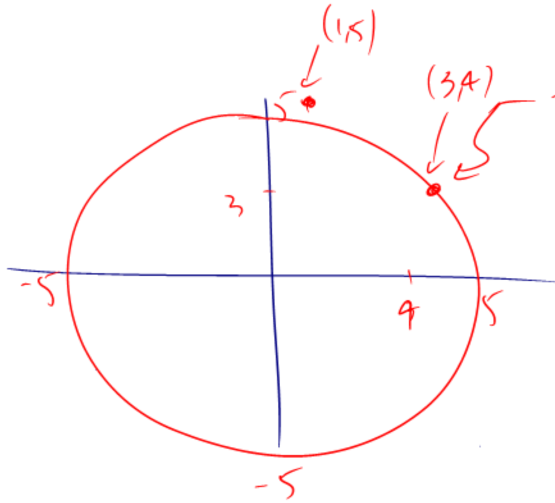
Hwk Check

Pg 13 #10

a)



$x^2 + y^2 = 5^2$

$\Rightarrow x^2 + y^2 = 25$

Test $(3,4)$

LHS $(3)^2 + (4)^2$

$= 25 = \text{RHS}$ ∴ yes.

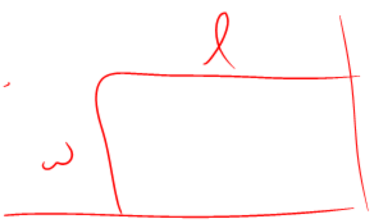
b) Test $(1,5)$

$\text{LHS} = (1)^2 + (5)^2$

$= 26 \neq 25$ ∴ No

c) VLT fails.

#6.



a) It's in the question.

b) $l + w = 12$

c) x

d) $2w + w = 12$

$w = 4\,m \Rightarrow l = 8\,m$

1.2 Properties of Functions

Recall that we define the graph of a function $f(x)$ to be the SET of Ordered Pairs:

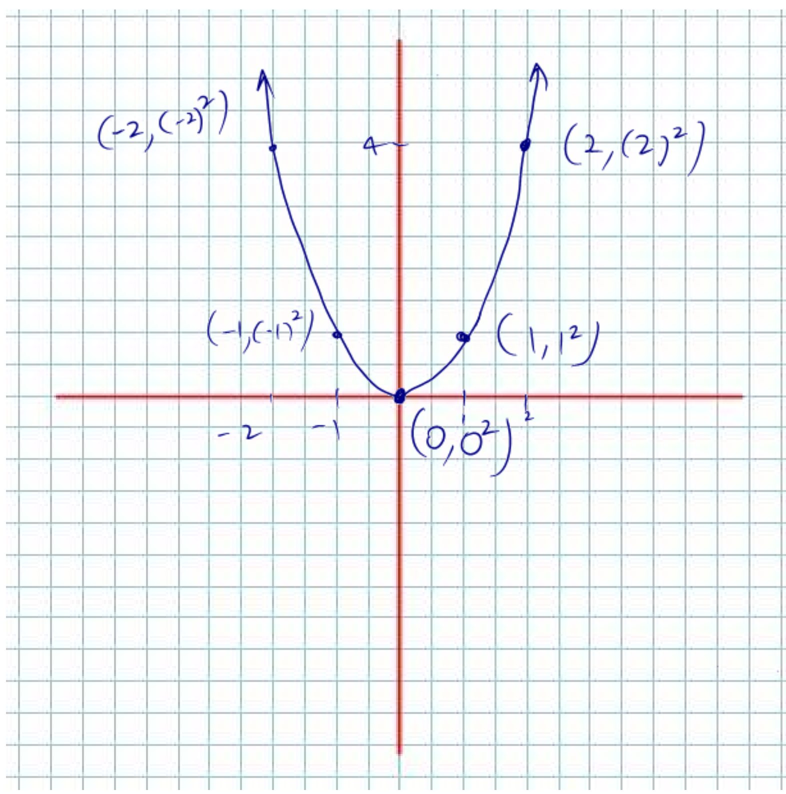
$$\{ (x, f(x)) \mid x \in D_f \}$$

We can visualize the graph of a function by plotting its ordered pairs on the Cartesian axes.

Example 1.2.1

e.g. $f(x) = x^2$ has the graph $\{ (x, x^2) \mid x \in \mathbb{R} \}$

and looks like



Characteristics of a Function's Graph

↳ Behaviours

Over the course we will be studying Polynomial, Rational, Trigonometric, Exponential and Logarithmic Functions. For now we are focussed on Polynomial and Rational Functions, but for each type of function we will try and understand various functional (fnal) behaviours (or characteristics).

The characteristics (behaviours) we are primarily interested in studying are:

1. Domain and Range
2. Axis Intercepts
3. Intervals of fnal increase and decrease on the domain
4. Odd vs Even fns (Symmetry)
5. Continuity vs discontinuity
6. Fnal End Behaviours

> Note: *Generally a geometric point of view will just mean that we'll look at pictures, but Geometry is actually* ***much*** *deeper than that!*

Intervals of Increase and Decrease

We will examine (when possible) functional behaviour from both algebraic and geometric points of view.

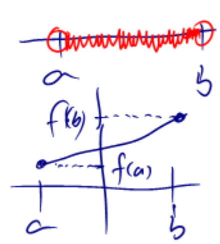
Definition 1.2.1

end points not included

domain values

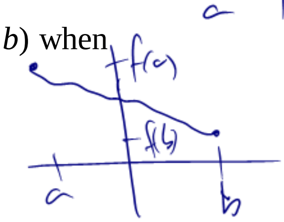
A function $f(x)$ is said to be increasing on the open interval (a,b) when

$f(a) < f(b)$ whenever $a < b$



A function $f(x)$ is said to be decreasing on the open interval (a,b) when

$f(a) > f(b)$ whenever $a < b$



Note the difference between open and closed intevals:

An open interval does not contain the end points

A closed interval contains the endpoints

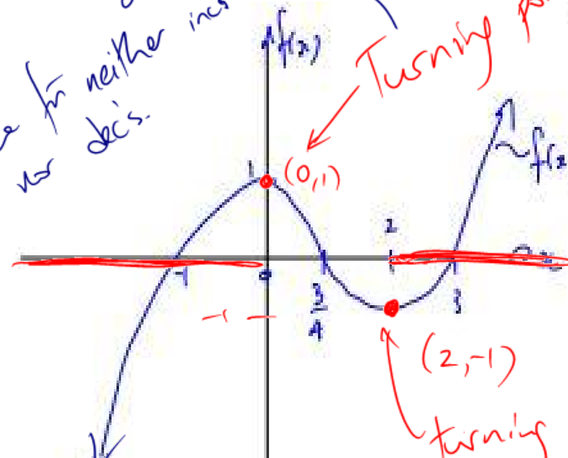
Example 1.2.2

Consider the function $f(x)$, represented graphically:

Determine where $f(x)$ is increasing and decreasing.

at turning pts the fn neither inc's nor dec's.

Turning points



turning pt

Figure 1.2.2

$f(x)$ is increasing on

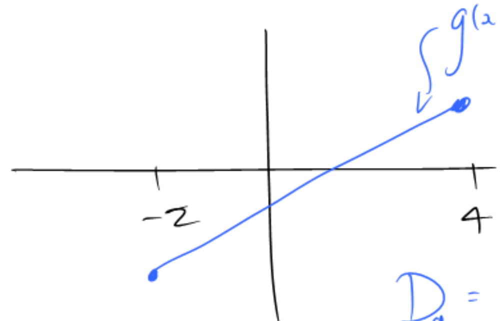
$(-\infty, 0) \cup (2, \infty)$ 'o' is not included

$f(x)$ is decresing on

$(0, 2)$

Consider the following:

g(x) is a line (segment)



$D_g = [-2, 4]$

$g(x)$ is increasing on

$[-2, 4]$ ($g(x)$ has no turning point)

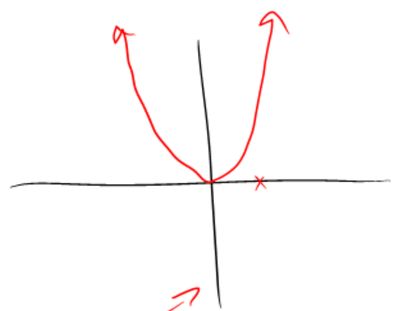
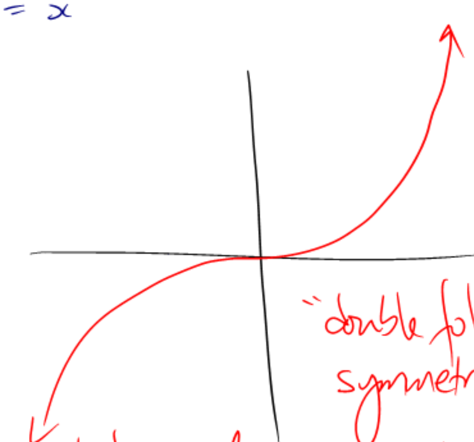
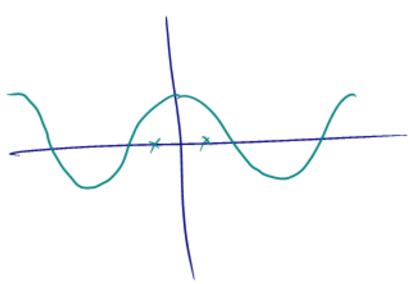
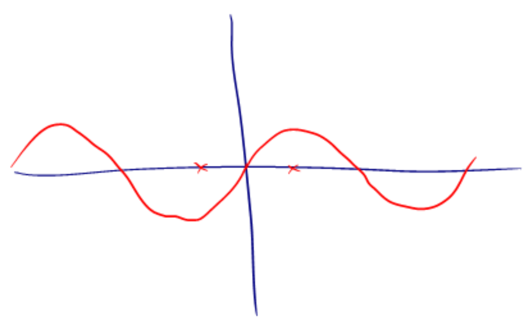
Odd vs. Even Functions

Note: This functional behaviour deals with SYMMETRY rather than the “power(s)” that you might see in various terms of the function.

Basic Definitions:

- Even Functions are symmetric around the final axis (the y axis)
- Odd Functions are symmetric around the origin

Graphical point of view:

Even Functions	*Odd Functions*
$f(x) = x^2$  final axis acts like a mirror	$f(x) = x^3$  "double fold" symmetry or rotation of 180° around the origin
$h(x) = \cos(x)$ 	$p(x) = \sin(x)$ 

Algebraically we will consider definitions for Even and Odd Functions:

Definition 1.2.2

A function $f(x)$ is **even** if $f(-x) = f(x)$ for every $x \in D_f$

A function $f(x)$ is **odd** if $f(-x) = -f(x)$ $\forall x \in D_f$

Algorithm
① Consider $f(-x)$
② Do some math
③ Compare the result w/ $f(x)$: $-f(x)$

Example 1.2.3

a) Show $f(x) = 3x^4 + 2x^2 + 5$ is even.

Consider $f(-x) = 3(-x)^4 + 2(-x)^2 + 5$
$= 3x^4 + 2x^2 + 5$
$= f(x)$ $\therefore f(x)$ is even

$(-x)^3 = -x^3$

b) Show $g(x) = 5x^3 - 2x$ is odd.

Consider $g(-x) = 5(-x)^3 - 2(-x)$
$= -5x^3 + 2x$ This is not $g(x)$! (factor out ~~a~~ neg)
$= -(5x^3 - 2x)$
$= -g(x)$ $\therefore g(x)$ is odd

c) Are i) $f(t) = 5t^3 - 2t + 1$ and

ii) $h(x) = \dfrac{3x^3 - 2x}{x^2 - 1}$ odd or even?

i) $f(t) = 5t^3 - 2t + 1$

Consider $f(-t) = 5(-t)^3 - 2(-t) + 1$

$= -5t^3 + 2t + 1$

$= -(5t^3 - 2t - 1)$

$\therefore$ $f(t)$ is neither odd nor even

ii) Consider

$h(-x) = \dfrac{3(-x)^3 - 2(-x)}{(-x)^2 - 1}$

$= \dfrac{-3x^3 + 2x}{x^2 - 1}$

$= -\left(\dfrac{3x^3 - 2x}{x^2 - 1}\right)$

$= -h(x)$

$\therefore$ $h(x)$ is odd

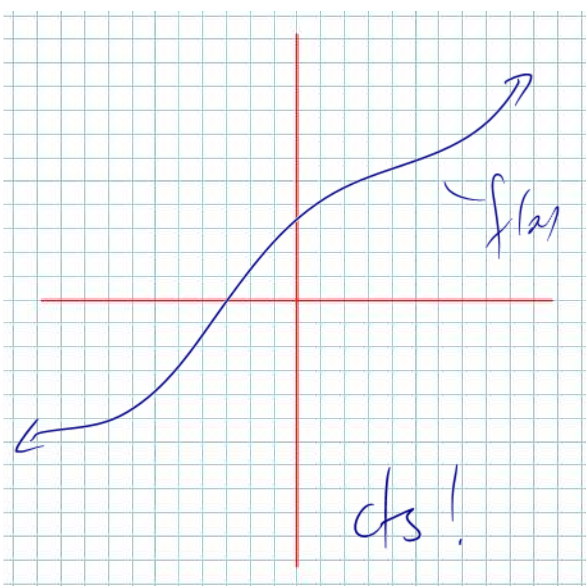
Continuity

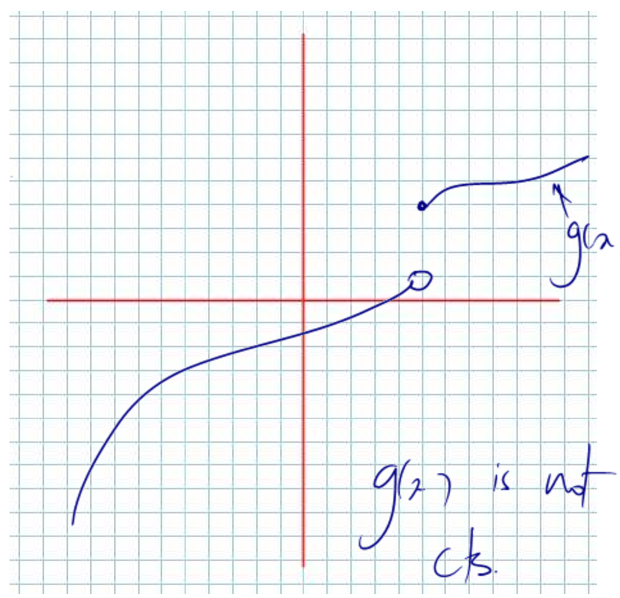
For the time being we will consider a (quite) rough definition of what it means for a function to be continuous. In fact, we will see that understanding what it means for a function to be discontinuous may be more helpful for now. In the course ***Calculus and Vectors***, a formal, algebraic definition of continuity will be considered.

Rough Definition

A function $f(x)$ is **continuous** (cts) on its domain D_f if when sketching the graph there is no need to lift pen from paper.

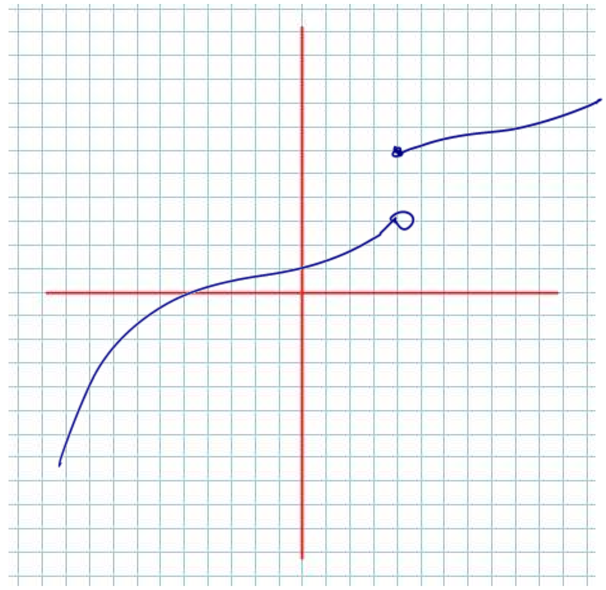
Pictures



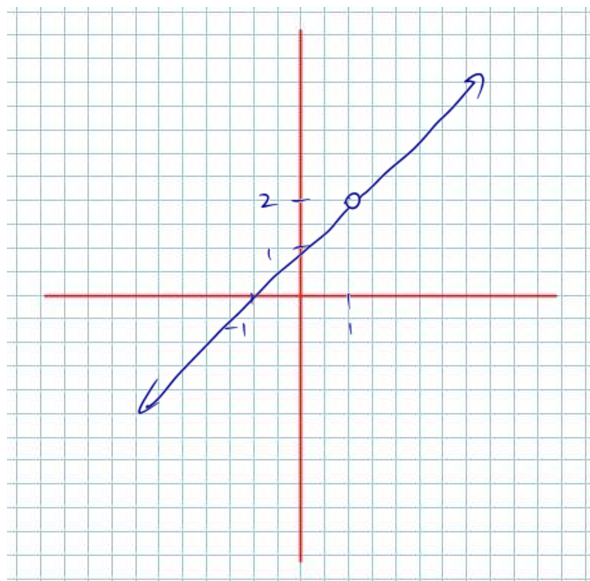


There are 3 types of **dis**continuities:

1) Jump

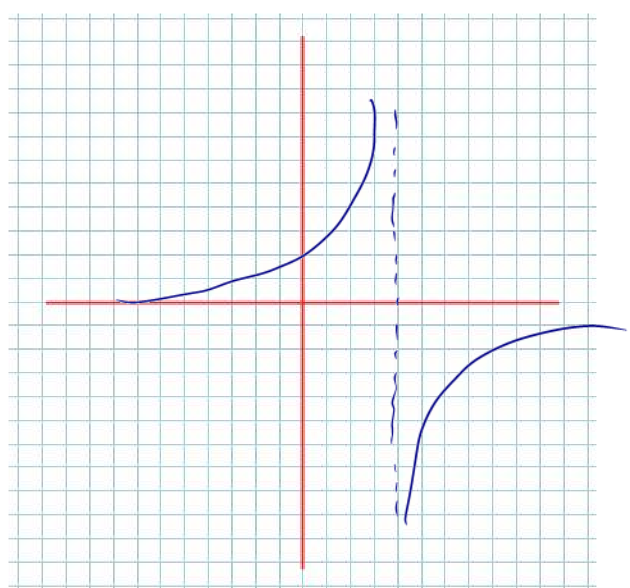


2) Hole (removable)



$f(x) = \dfrac{x^2-1}{x-1}$

3) Infinite (Asymptotic)



Hwk Check

pg 23 #5b $f(x) = \sin(x) + x$

$\sin(x)$ is <u>odd</u>

$\therefore \sin(-x) = -\sin(x)$

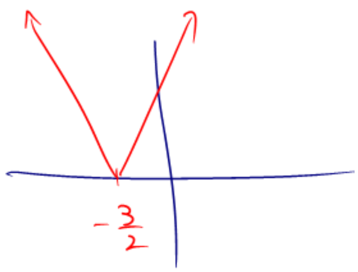
Consider $f(-x) = \sin(-x) + (-x)$

$= -\sin(x) - x$

$= -(\sin(x) + x)$

$= -f(x) \Rightarrow f(x)$ is odd

f) $f(x) = |2x+3|$



Consider $f(-x) = |2(-x)+3|$

$= |-2x+3|$

$= |-(2x-3)|$

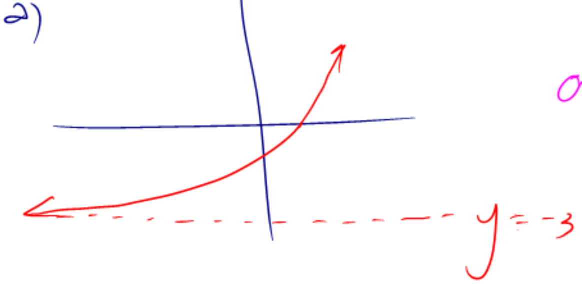
$= |-1|\,|2x-3|$

$= |2x-3|$ not $f(x)$

$|a \cdot b| = |a| \cdot |b|$

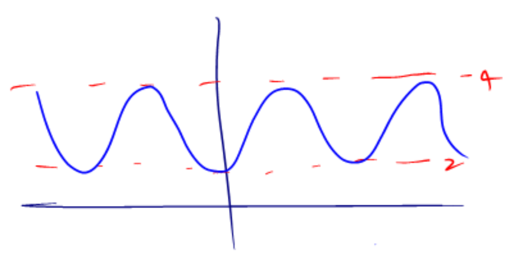
$\therefore$ not odd nor even

#8 a)

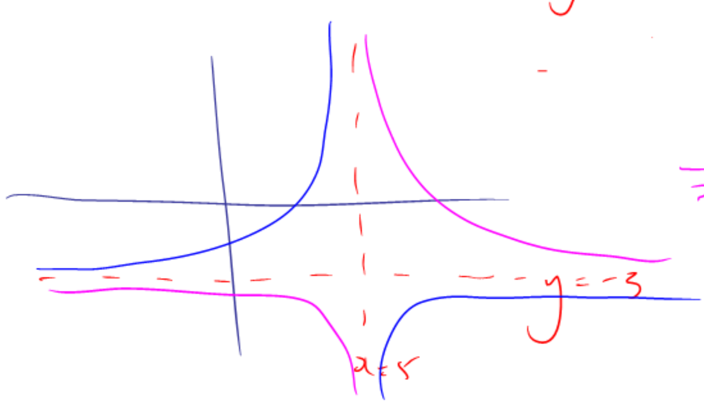


a^x

b)



$\cos(x)$



$\frac{1}{x}$

End Behaviour of Functions

Here we are concerned with how the function is behaving as x gets HUGE

As x gets HUGE (which we write $x \to \infty$, or $x \to -\infty$)

"tends to"

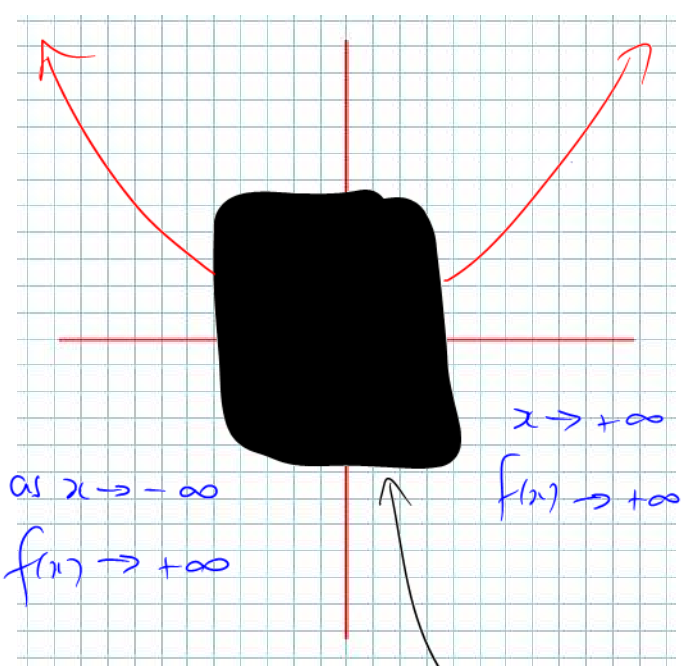
the functional values (for whatever function we are studying) can do one of three things:

1) Go to infinity — e.g. as $x \to \pm\infty$, $f(x) \to \pm\infty$

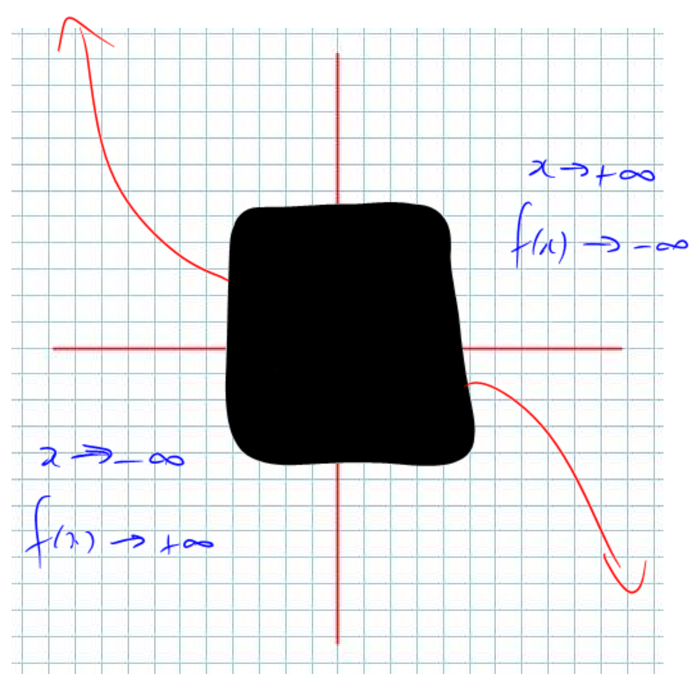
2) Settle down to single value — horizontal asymptote: $x \to \pm\infty$, $f(x) \to a$ (a is a finite #)

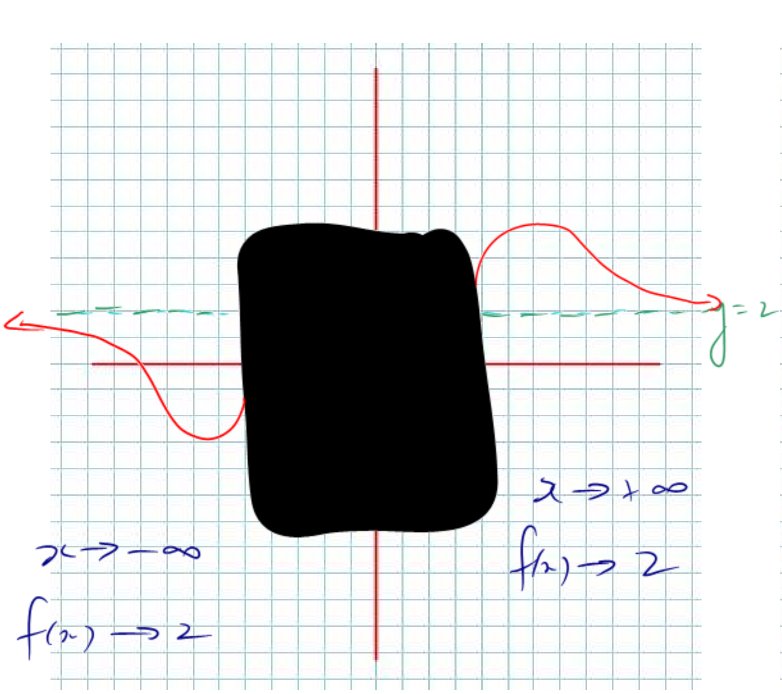
3) Oscillate (bounce) between 2 values

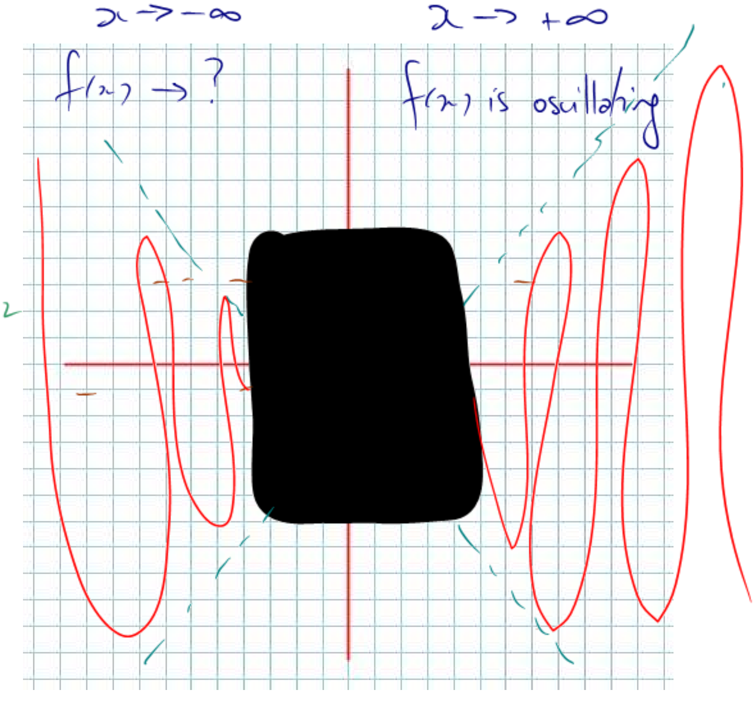
Pictures:



big black box of mystery







Class/Homework for Section 1.2

Pg. 23 – 24 #5, 7 - 11 #5, 7, 8