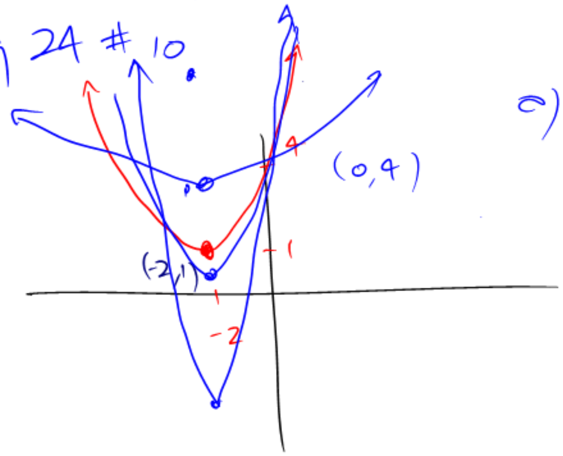
Hwk Check

Patrick - 5003.

pg 24 # 10



a)

$$f(x) = a(x-h)^2 + k$$

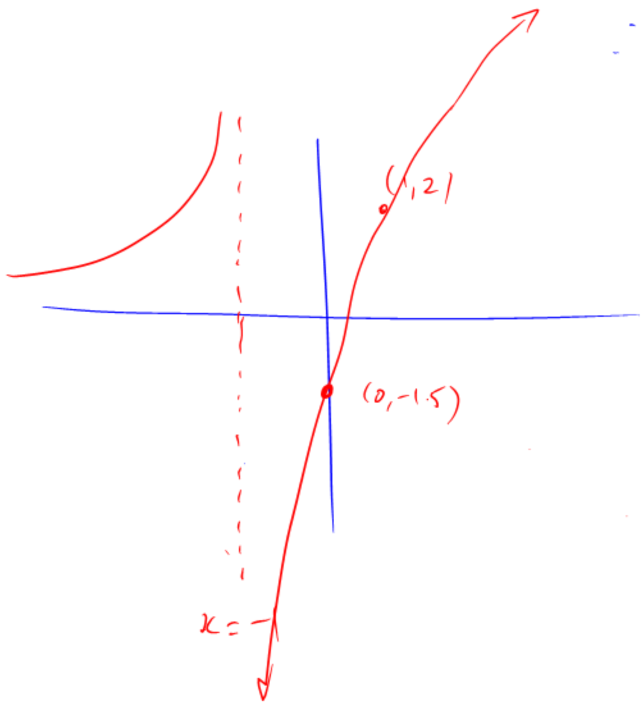
$$f(x) = a(x+2)^2 + 1$$ use (0,4) to find a

$$4 = a(4) + 1$$

$$\Rightarrow 3 = 4a \Rightarrow a = \frac{3}{4}$$

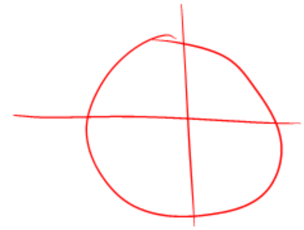
$$\therefore f(x) = \frac{3}{4}(x+2)^2 + 1$$

9.



1.4 Inverses of Functions

The inestimable William Groot has a saying:



An Inverse *Relation* is an UNDO

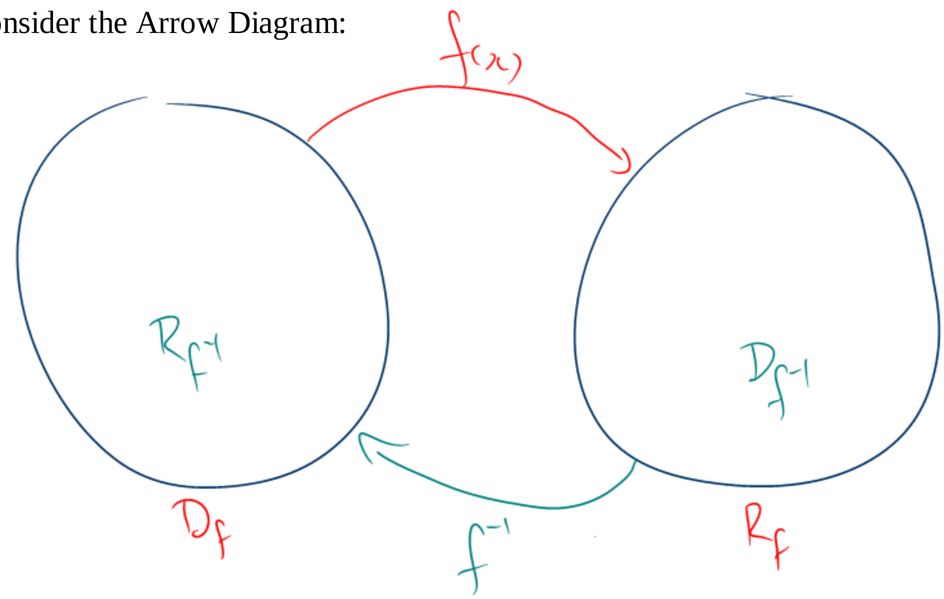
Definition 1.4.1

A **relation** is simply an algebraic relationship between domain values and range values.

Note: All functions are relations, but not all relations are functions

e.g. $x^2 + y^2 = 25$ is a relation, but it is not a function (it's a circle and so doesn't pass the VLT)

Consider the Arrow Diagram:



↑ the inverse of f(x)

Note: $f^{-1}(x)$ may not be a fn

Big Concept

To get an inverse one must switch "x & y" (domain & range)

Example 1.4.1

Given the graph of $f(x)$ determine: $D_f, R_f, f^{-1}(x), D_{f^{-1}}, R_{f^{-1}}$

$f^{-1}(x)$ is not a fn

$$f(x) = \{(2,3),(4,2),(5,6),(6,2)\}$$

$$f^{-1}(x) = \{(3,2),(2,4),(6,5),(2,6)\}$$

$$D_f = \{2,4,5,6\}$$

$$R_f = \{3,2,6\}$$

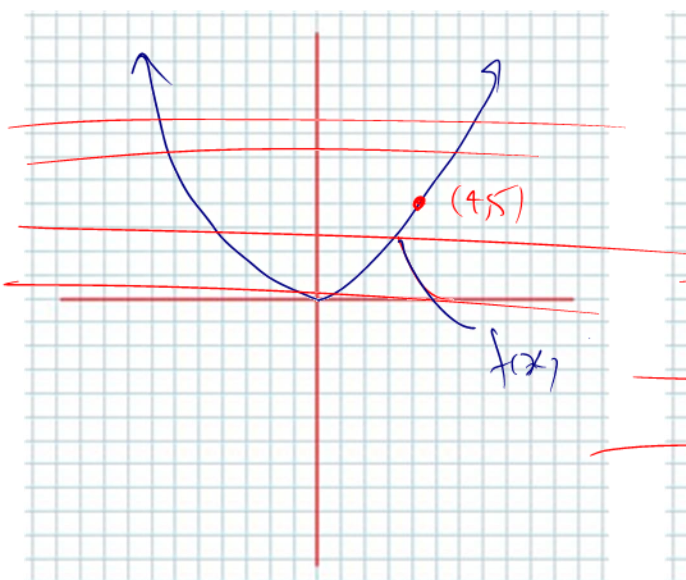
$$D_{f^{-1}} = \{3,2,6\} = R_f$$

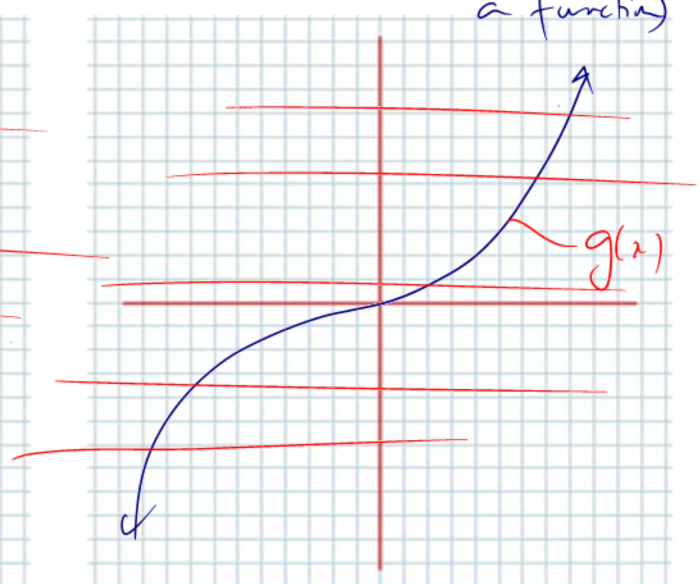
$$R_{f^{-1}} = \{2,4,5,6\} = D_f$$

Horizontal Line Test

(like the VLT BUT it tells us if a fn's inverse is a function)

Consider the Sketches





Because the HLT fails $f^{-1}(x)$ is not a fn!

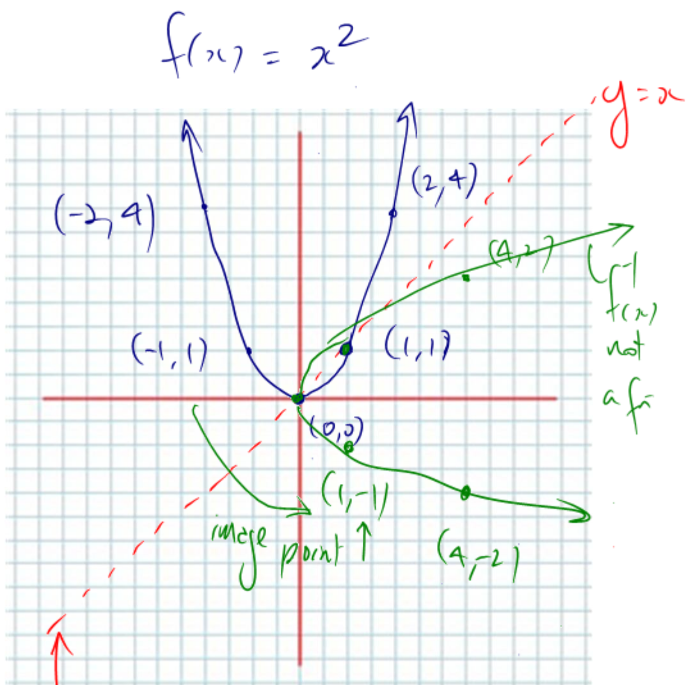
HLT passes $\Rightarrow g^{-1}(x)$ is a fn

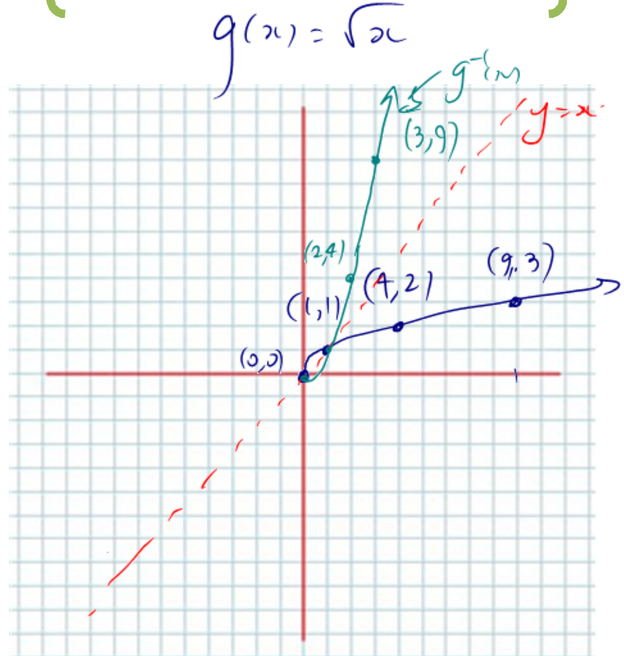
Determining the Inverse of a Function

We can determine the inverse of some given function in either of two ways: Graphically and Algebraically.

Note: Finding a function inverse graphically is not a very useful method, but it can be instructive.

Function Inverses Graphically





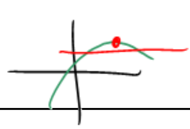
every point on $y = x$ is invariant under an inversion and the line $y = x$ is a "mirror"

Note: $g^{-1}(x)$ is half a parabola.

If $f(x)$ had been on a restricted domain $([0, \infty))$ then $f^{-1}(x)$ would have been a fn

Restricting the Domain

Turning points are the problem. Around turning points a fn will FAIL the HLT!



Function Inverses Algebraically

Determining algebraic representations of inverse relations for given functions can be done in (at least) two ways:

1) Use algebra in a "brute force" manner (keeping in mind the Big Concept)
2) Use Transformations (keeping in mind "inverse operations")

Example 1.4.2

Determine the inverse of $f(x) = 2\sqrt{\frac{1}{3}(x-1)} + 2$.

State the domain and range of both the function and its inverse.

Here we will use "brute force".
Method:

1) Switch x and $f(x)$, and call "$f(x)$", $f^{-1}(x)$.
2) Solve for $f^{-1}(x)$

$D_f = [1, \infty)$

$R_f : [2, \infty)$

$$x = 2\sqrt{\tfrac{1}{3}(f^{-1}(x) - 1)} + 2$$

$$\Rightarrow x - 2 = 2\sqrt{\tfrac{1}{3}(f^{-1}(x) - 1)}$$

$$\Rightarrow \tfrac{1}{2}(x-2) = \sqrt{\tfrac{1}{3}(f^{-1}(x) - 1)}$$

$$\Rightarrow \left(\tfrac{1}{2}(x-2)\right)^2 = \tfrac{1}{3}(f^{-1}(x) - 1)$$

$$\Rightarrow 3\left(\tfrac{1}{2}(x-2)\right)^2 = f^{-1}(x) - 1$$

$$\boxed{\therefore f^{-1}(x) = 3\left(\tfrac{1}{2}(x-2)\right)^2 + 1}$$

$D_{f^{-1}} = [2, \infty)$

$R_{f^{-1}} : [1, \infty)$

Example 1.4.3

Using transformations determine the inverse of $f(x) = \overset{V_D}{2}\sqrt{\underset{H_D}{\frac{1}{3}}(x\underset{H_T}{-1})} + \overset{V_T}{2}$.

$$f^{-1}(x) = 3\left(\tfrac{1}{2}(x-2)\right)^2 + 1$$

negatives stick w/ their #'s

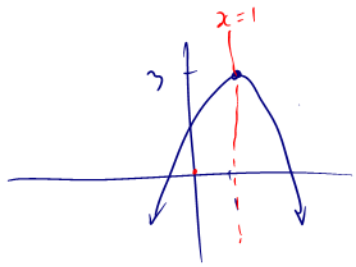
Example 1.4.4

① Determine the inverse of $g(x) = -2(x-1)^2 + 3$.

② Note that the natural domain of $g(x)$ is $(-\infty, \infty)$. However, $g(x)$ does not pass the HLT so its inverse is not a function. Determine a restricted domain for $g(x)$ so that $g^{-1}(x)$ ***is*** a function.

① $g^{-1}(x) = \sqrt{-\frac{1}{2}(x-3)} + 1$

②



On the interval $[1, \infty)$

or $(-\infty, 1]$

Example 1.4.5

Given $f(x) = kx^2 - 3$ and given $f^{-1}(5) = 2$, find k.

Two methods:

① $f^{-1}(x) = \sqrt{\frac{1}{k}(x+3)}$

use (5,2) to find k

$\Rightarrow 2 = \sqrt{\frac{1}{k}(5+3)}$

$4 = \frac{1}{k}(8)$

$\Rightarrow 4 = \frac{8}{k}$

$k = \frac{8}{4}$

$= 2$

(5,2) is a pt on f^{-1}

$\Rightarrow$ (2,5) is on $f(x)$

$5 = k(2)^2 - 3$

$8 = 4k$

$k = 2$

Class/Homework for Section 1.4

Pg. 43 – 45 #2 – 4, 7, 9, 12, 13, 15