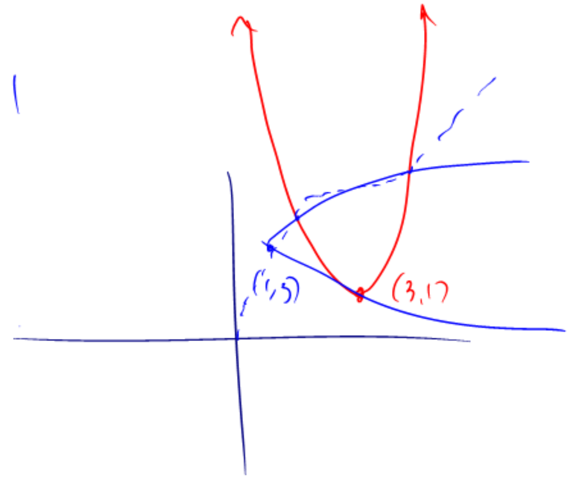
Hwk Check

Pg 45 #13 $\quad g(x) = 4(x-3)^2+1$

a) $g^{-1}(x) = \pm\sqrt{\frac{1}{4}(x-1)} + 3$



e) $[3, \infty)$ or $(-\infty, 3]$

f) Not a fn because the turning point $(3,1)$ is inside $[2, 5]$

1.5 Piecewise Defined Functions

Some aspects of “reality” exhibit different (as opposed to changing) $\boxed{\text{Behaviours}}$

To capture those different behaviours mathematically may require using different $\boxed{\text{function}}$ over different $\boxed{\text{parts / pieces}}$ of the domain.

Absolute Value §1.2

Before discussing piecewise defined functions in general, we will first review the concept of *absolute value*.

Definition 1.5.1

The absolute ***value*** of a number, *x*, is given by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$|-10| = -(-10)$$
$$= +10$$

e.g.’s

$|22| =$ 22

$|-8| =$ 8

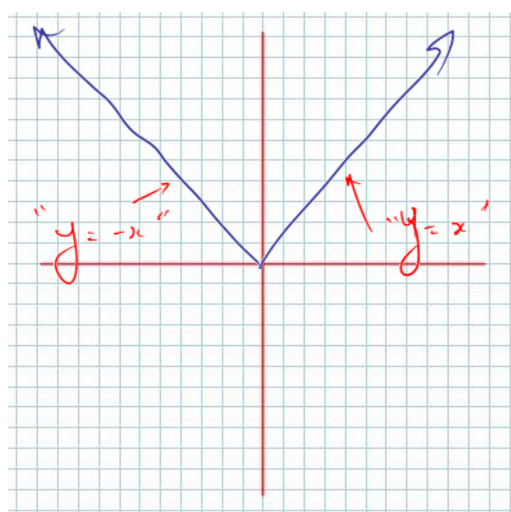
$|8-13| =$ $|-5| = +5$

Absolute Value Functions

We can define the function which returns the absolute value for any given number as

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Picture



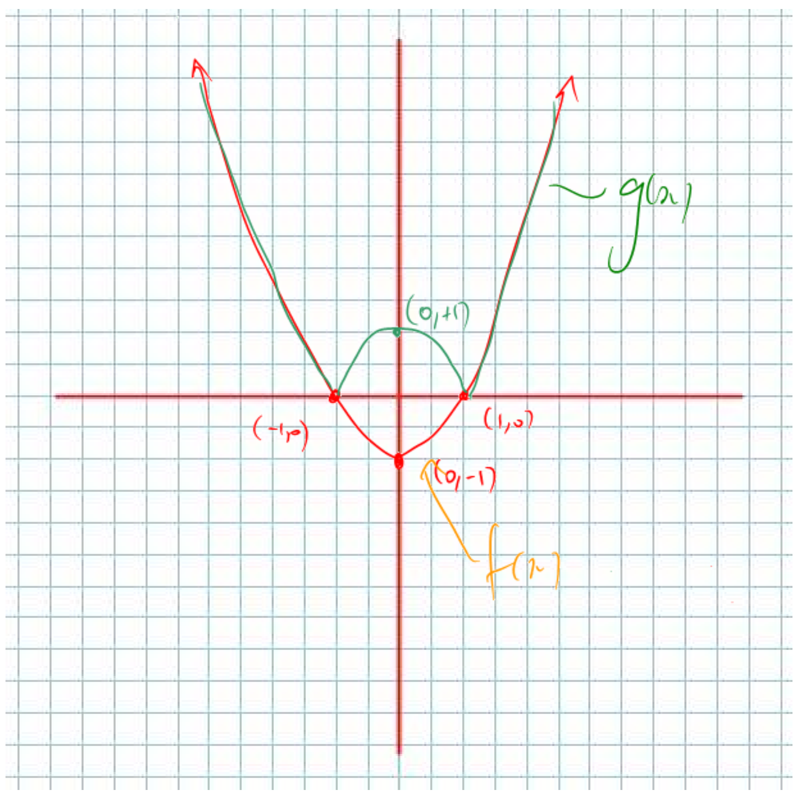
(Two behaviours!)

① On $[0, \infty)$ $f(x)$ is behaving like "$y = x$"

② On $(-\infty, 0]$, $f(x)$ is behaving like "$y = -x$"

We can go further and define functions which return the absolute value for more complicated expressions.

e.g. Sketch $g(x) = |x^2 - 1|$ (note: g(x) takes the absolute value of the ***functional values*** for the "basic" function $f(x) = x^2 - 1$)



on $(-\infty, -1]$

on $[-1, 1]$

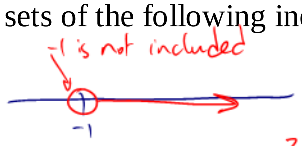
on $[1, \infty)$

(Three functional behaviours)

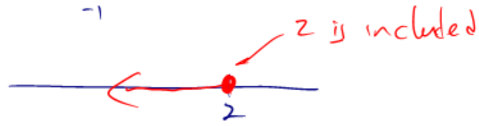
Absolute Value and Domain Intervals (and Quadratic Inequalities)

e.g.'s Sketch the solution sets of the following inequalities:

a) $x > -1$

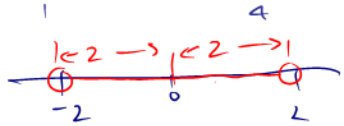


b) $x \le 2$



c) $1 < x \le 4$

d) $-2 < x < 2$



have same abs val

Note the symmetry in part d)! Sometimes it's useful to think of absolute value as

$\boxed{\text{distance from the origin.}}$

Using the above notion we can thus use absolute value to denote the interval $-2 < x < 2$ as

$$|x| < 2$$

e.g. Solve the quadratic equation

$x^2 = 4$ $\quad x = \pm 2 \Rightarrow |x| = 2$

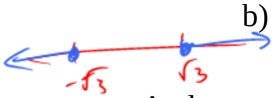
e.g. Solve the quadratic inequalities, and sketch the solution sets:

same inequality direction

a) $x^2 < 4$ $\quad |x| < 2 \Rightarrow -2 < x < 2$

rooting a variable give abs val

b) $x^2 \geq 3$ $\quad |x| \geq \sqrt{3} \Rightarrow -\sqrt{3} \geq x \geq \sqrt{3}$

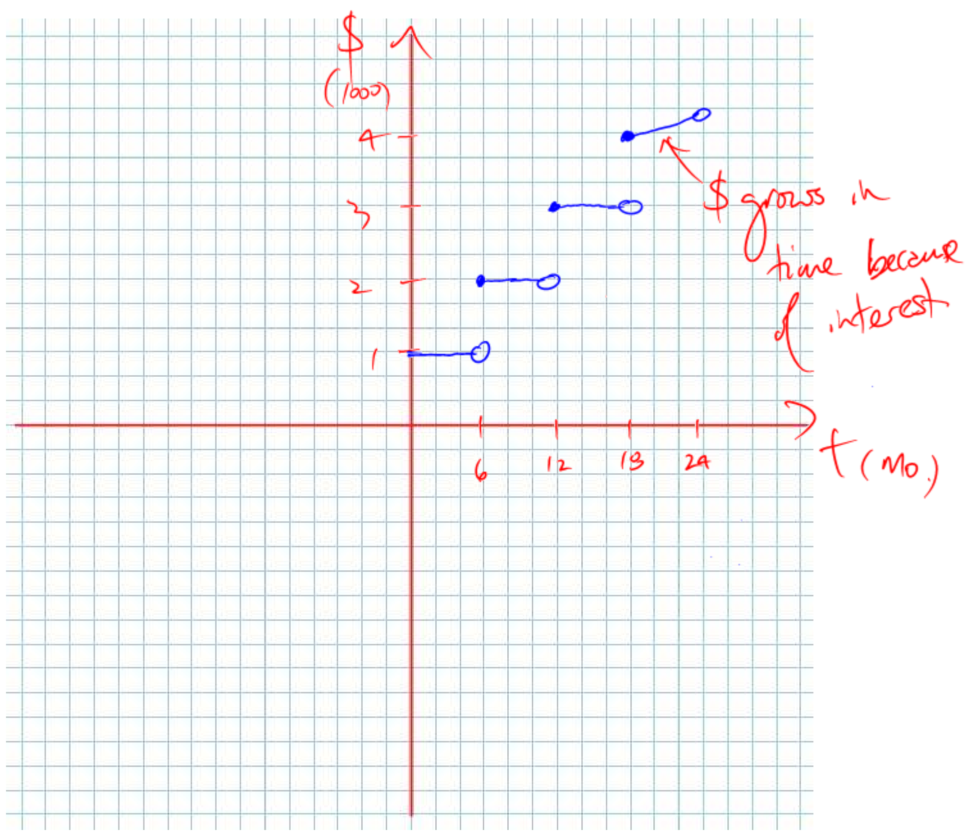


And now we return our attention to general **Piecewise Defined Functions**

Example 1.5.1

You are saving for university, and place \$1000 into a sock every six months. After 18 months you wake up and put the money in your sock into an interest bearing bank account. You continue making deposits. Give a graphical representation of this situation.

What is the behaviour of the amount of money you have saved? How is the behaviour changing?



Example 1.5.2

Determine the graphical representation for:

$$f(x)=\begin{cases} x^2, & x\in[-1,2) \\ 2, & x\in[2,3] \\ x+1, & x\in(3,\infty) \end{cases}$$

Note the notation we use for piecewise defined functions. Each functional behaviour has a mathematical representation, defined over its own piece of the domain (just like the Absolute Value function we considered earlier.

$D_f = [-1, \infty)$

$f(-1) = 1$

$f(0) = 0$

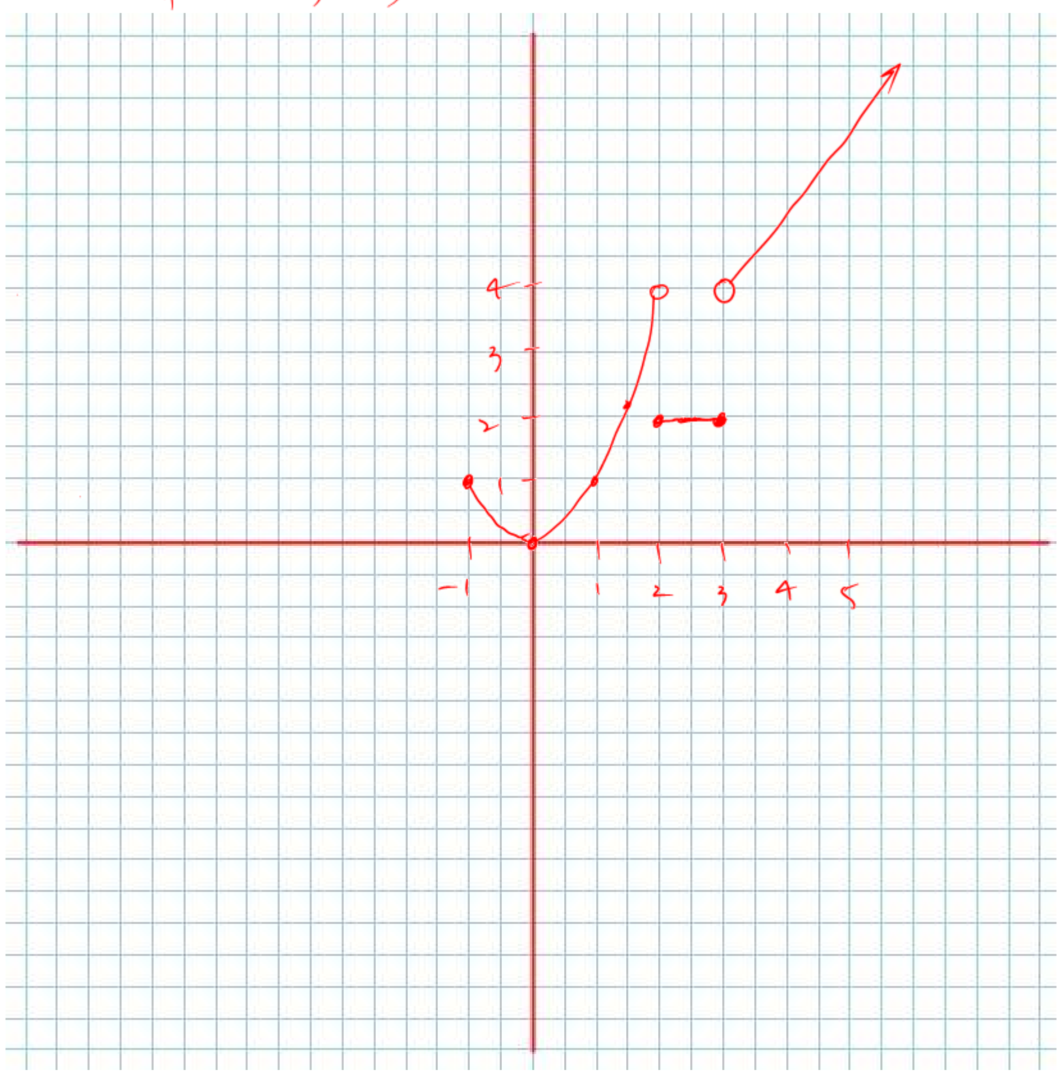
$f(1) = 1$

$f(2) = 2$

$f(2.5) = 2$

$f(3) = 2$

$f(4) = 5$



3 behaviors!

Example 1.5.3

Determine a possible algebraic representation which describes the given functional behaviour.

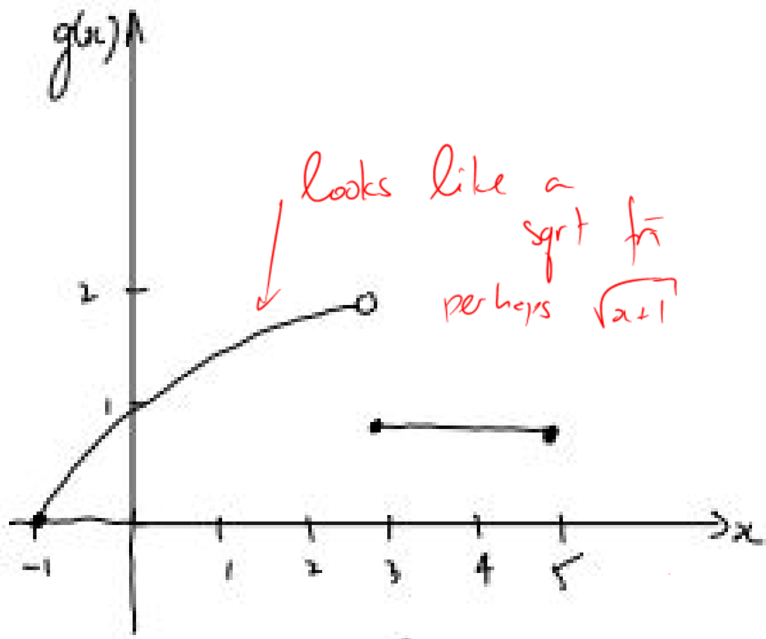


Figure 1.5.3

Looks like a sqrt fn perhaps $\sqrt{x+1}$

Two behaviours
on $[-1,3)$ we have "$\sqrt{\ }$"
on $[3,5]$ we have a constant fn

on $[-1,3)$

$$g(x) = \begin{cases} \sqrt{x+1}, & -1 \le x < 3 \\ 1, & 3 \le x \le 5 \end{cases}$$

> *Class/Homework for Section 1.5*
>
> *(Abs. Value.) Pg. 16 #2, 4 – 8 (think about transformations!), 10*
> *(Piecewise) Pg. 51 – 53 #1 – 5, 7 – 9*