1.6 Combinations of Functions

By now you should have a pretty good sense of how to combine numbers.

$$\text{e.g. } 3+5,\ 7\div 4,\ \text{or } 3.9\times\frac{4}{3}\ \text{etc.}$$

Functions can be thought of as number generators, and if numbers can be combined, then in the same way (using the basic algebraic operations) we should be able to combine functions too.

A **BIG QUESTION** to ask is:

can mean any mathematical operation

$+, -, \times, \div$

What does it mean to $*$ two (or more) fns together?

A **BIGGER QUESTION** to ask is:

If fns are number generators using domain value, upon what does $*$ act?

Going back to basic graphs of functions may prove helpful in understanding what's happening here.

Notation:

$(f * g)(x)$ means $f(x) * g(x)$

Example 1.6.1

Given the functions

$D_{f+g} = \{-1, 0, 1\}$

$$f(x) = \{(-1,2), (0,5), (1,\pi), (2,-4)\}$$

$$g(x) = \left\{(-2,1), (-1,-1), (0,5), \left(1, \frac{2\pi}{3}\right)\right\}$$

determine:

a) $f(x) + g(x)$

b) $g(x) \div f(x)$

c) $g(x) \times (-f(x))$

a) $(f+g)(-1) = f(-1) + g(-1) \Rightarrow (-1, 1)$
$= 2 + (-1) = 1$

$(f+g)(0) = f(0) + g(0) = 5 + 5 = 10 \Rightarrow (0, 10)$

$(f+g)(1) = f(1) + g(1) = \pi + \frac{2\pi}{3} = \frac{5\pi}{3} \Rightarrow (1, \frac{5\pi}{3})$

$(f+g)(2) = f(2) + g(2) = -4 + ?$

a) $f(x) + g(x) = \{(-1,1), (0,10), (1, \frac{5\pi}{3})\}$

b) $(g \div f)(x) = g(x) \div f(x)$

eg $(g \div f)(-1) = g(-1) \div f(-1) = -\frac{1}{2} \Rightarrow (-1, -\frac{1}{2})$

c) $g(-1) \times (-f(-1)) = (-1) \times (-2) = 2 \Rightarrow (-1, 2)$

Note how the domain of the combined functions is determined by the domains of the original functions!

Definition 1.6.1

Given the functions $f(x)$ and $g(x)$ with domains D_f and D_g respectively, then the domain of the combined function $(f * g)(x)$ is given by:

$D_{f*g} = D_f \cap D_g$ intersection

Caveat for $D_{f \div g} = D_f \cap D_g$, $g(x) \neq 0$

(Note: The operation "$*$" could mean any of the basic algebraic operations)

Example 1.6.2

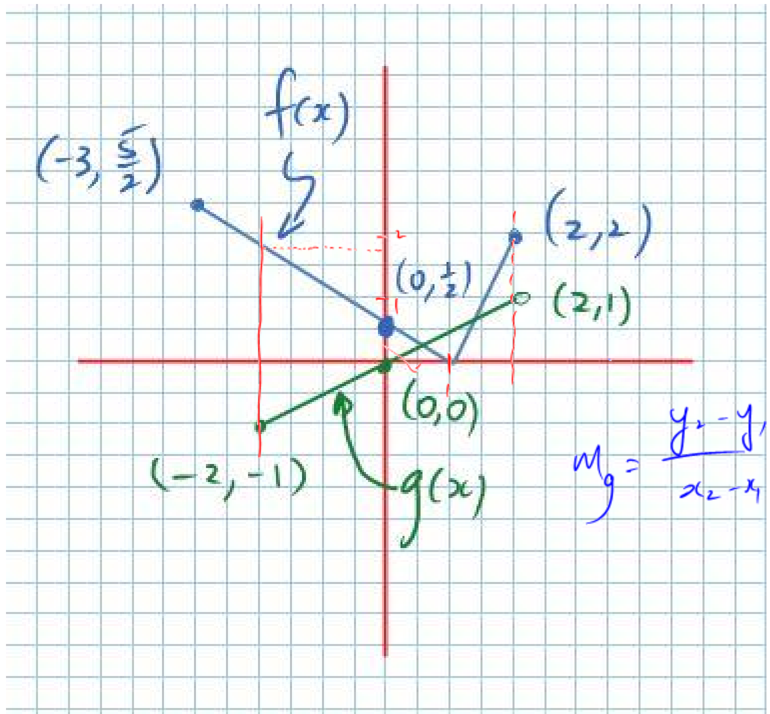
Given the sketches of the functions $f(x)$ and $g(x)$ determine graphically (giving both a rough sketch and a sample (at least 3 points) of the graph):

a) $f(x) - g(x)$
b) $f(x) \times g(x)$
c) $(g(x))^2$

$D_{f-g} = [-2, 2]$

$y_1 = m_1 x + b_1$
$y_2 = m_2 x + b_2$

$y_1 - y_2 = (m_1 x + b_1) - (m_2 x + b_2)$
$= (m_1 - m_2)x + (b_1 - b_2)$
$= mx + b$



$m_g = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2}$

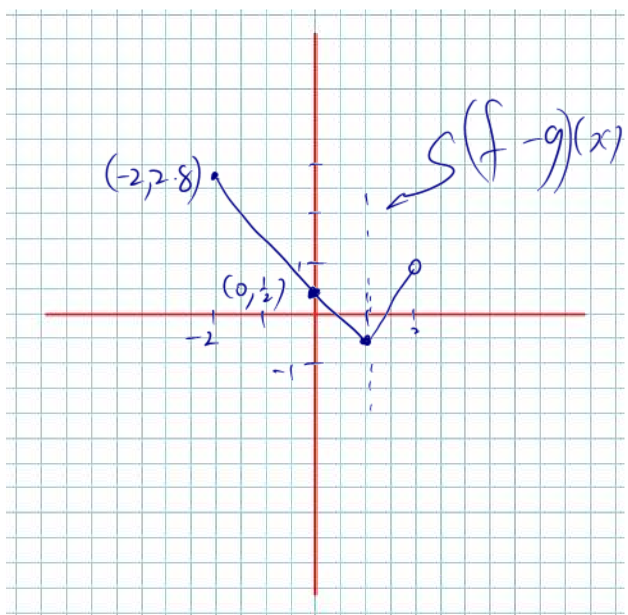
Figure 1.6.2

$(f-g)(-2)$
$= f(-2) - g(-2)$
$= 1.8 - (-1) = 2.8$

a)

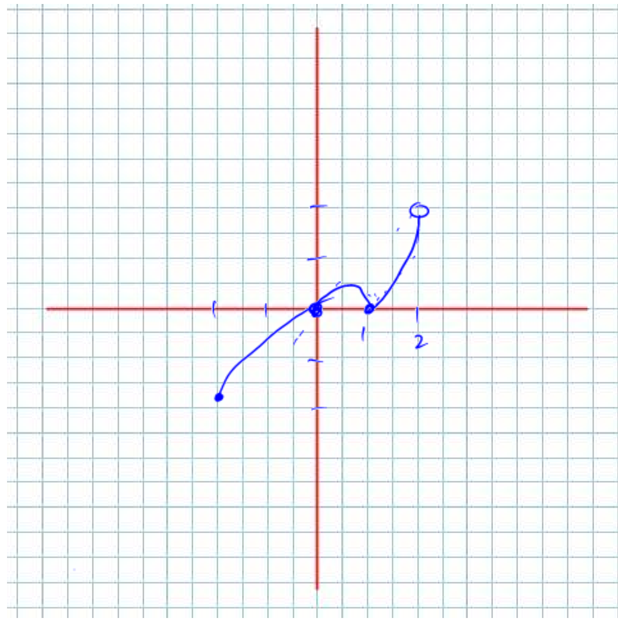
$f(0) - g(0) = \frac{1}{2} - 0 = \frac{1}{2}$

$f(1) - g(1) = 0 - \frac{1}{2} = -\frac{1}{2}$



$y_1 = m_1 x + b_1$
$y_2 = m_2 x + b_2$ $\Big\} \; y_1 \cdot y_2 = (m_1 x + b_1)(m_2 x + b_2) = m_1 m_2 x^2 + \ldots$

b) $f(x) \times g(x)$



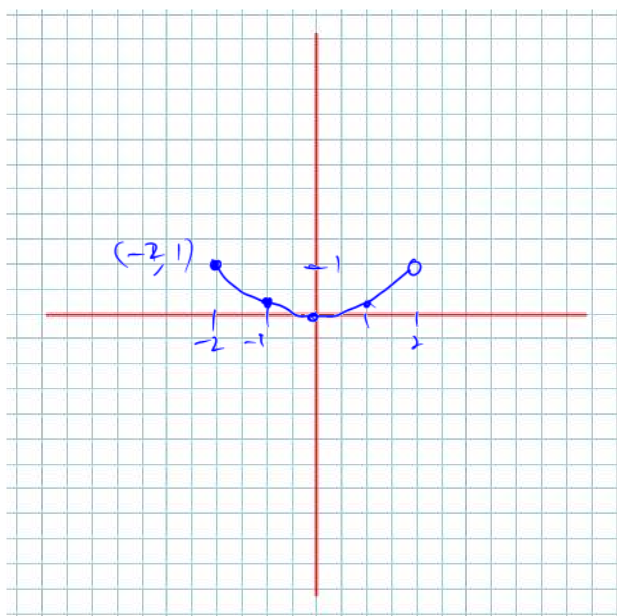
$f(-2) \times g(-2) = (1.8) \times (-1)$
$= (-1.8)$

$f(0) \times g(0) = (\frac{1}{2})(0) = 0$

$f(1) \times g(1) = (0)(\frac{1}{2}) = 0$

$f(2) \times$ " $g(2)$ " $= 2 \times$ " 1 " $= 2$

c) $(g(x))^2$



$g(x) = \frac{1}{2}x$

$= g(x) \cdot g(x)$

$= (\frac{1}{2}x)(\frac{1}{2}x)$

$= \frac{1}{4}x^2$

$g(-2) = \frac{1}{4}(-2)^2 = 1$

$g(-1) = \frac{1}{4}(-1)^2 = \frac{1}{4}$

Class/Homework for Section 1.6

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pg 62 #1, 2, 3, 5, 7, 10
+ inverses