Advanced Functions

Chapter 2 – Polynomial Functions

Fall 2013

Course Notes

(Note: *This material is taken from Chapter 3 in the textbook*)

Chapter 2 – Polynomial Functions

Contents with suggested problems from the Nelson Textbook (Chapter 3)

2.1 Polynomial Functions: An Introduction

Definition 2.1.1

A **Polynomial Function** is of the form

ellipsis : follow the pattern

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

where a_i, $i = 0, 1, 2, \dots, n$ are called coefficients, a_i are real numbers (but for us, a_i will usually be integers)

and "n" will be a Natural Number $\mathbb{N} = \{1, 2, 3, \dots\}$ (the naturals are the counting numbers)

Examples of Polynomial Functions

a) $f(x) = 8x^4 - 5x^3 + 2x^2 + 3x - 5$

$a_4 = 8$, $a_3 = -5$, $a_2 = 2$ $a_1 = 3$, $a_0 = -5$

b) $g(x) = 7x^6 - 4x^3 + 3x^2 + 2x$

There are zero "x^5's and x^4's"

$a_6 = 7$, $a_5 = 0$, $a_4 = 0$, $a_3 = -4$, $a_2 = 3$, $a_1 = 2$, $a_0 = 0$

terms are constructed by multiplying factors together, Terms are separated from each other by adding & subtracting

Notes: The **TERM** $a_n x^n$ in any polynomial function (where n is the **highest power** we see) is

called the LEADING TERM, and then we write all the following terms

in descending order.

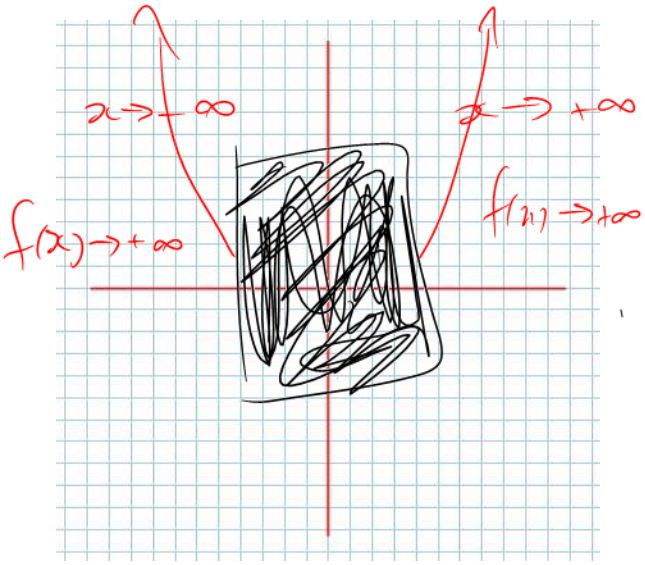
The Leading Term has two components:

1) Leading Coefficient a_n. Note a_n could be +ve or -ve

2) n. Note n can be even or odd

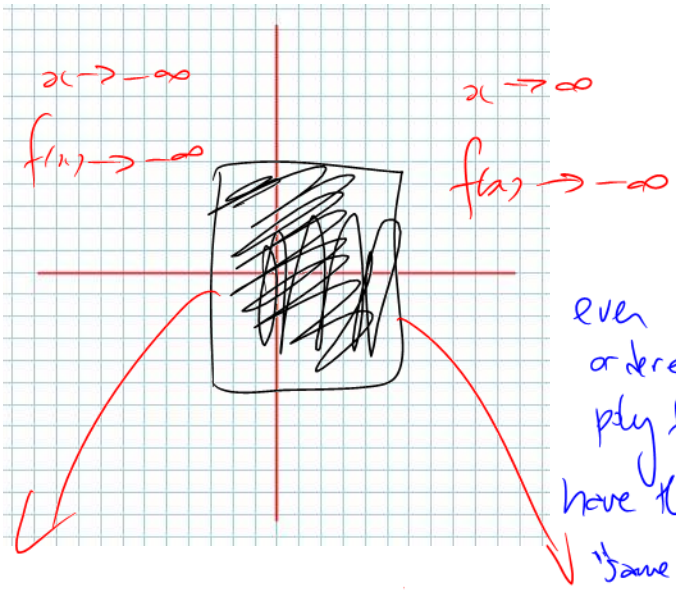
The Leading Term tells us the **end behaviour** of the polynomial function.

Pictures



$a_n > 0$

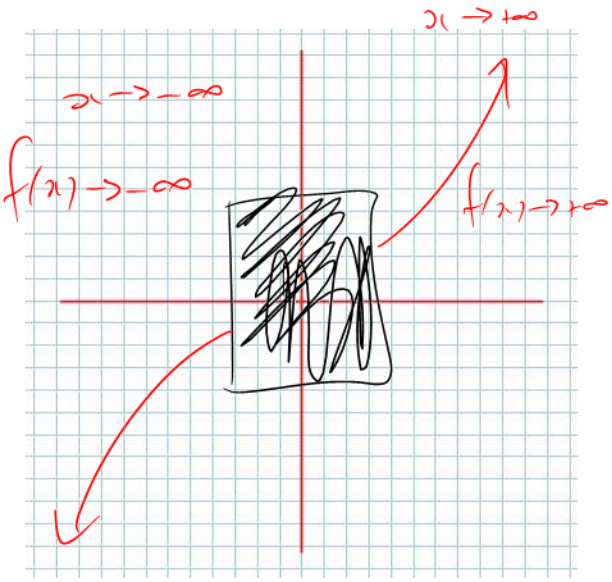
n even



even ordered poly fns have the "same end behaviours"

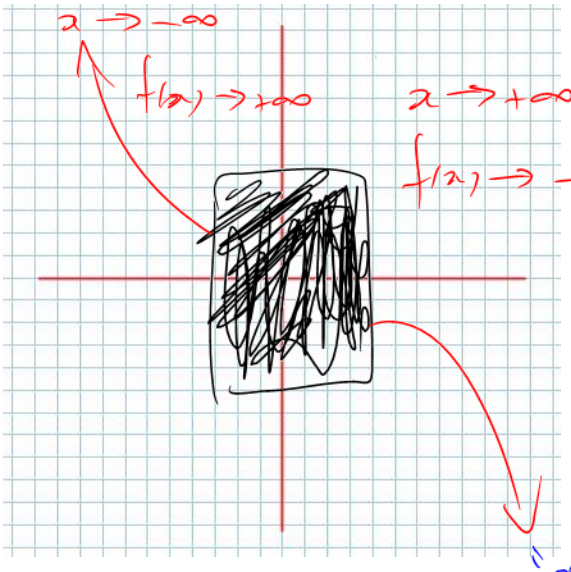
$a_n < 0$

n even



$a_n > 0$

n odd



odd ordered poly fns have "opposite end behaviours"

$a_n < 0$

n odd

Definition 2.1.2

The **order** (degree) of a polynomial is the value of the 'power' on the leading term.

eg $f(x) = -2x^8 - 5x^6 + 2x - 5$ is an order 8 (or degree 8) polynomial

Finite Differences (between successive functional values) can tell us the order of a polynomial:

x	$f(x)$	1ST F.D.	2ND F.D	3RD F.D	...
-1	-1				
		1			
0	0		0		
		1		6	
1	1		6		
		7		6	
2	8		12		
		19			
3	27				

Once you hit a constant finite difference that column gives the order of the polynomial fitting the "data". for above an order (3) polynomial describes flag

Class/Homework for Section 2.1

Pg. 122 #1 – 3 (Review on Quadratic Factoring)
Pg. 127 – 128 #1, 2, 3cd, 5, 6