Tie −35 000 for attitude.

2.2 Characteristics (Behaviours) of Polynomial Functions

Today we open, and look inside the black box of mystery

Consider the sketch of the graph of some function, $f(x)$:

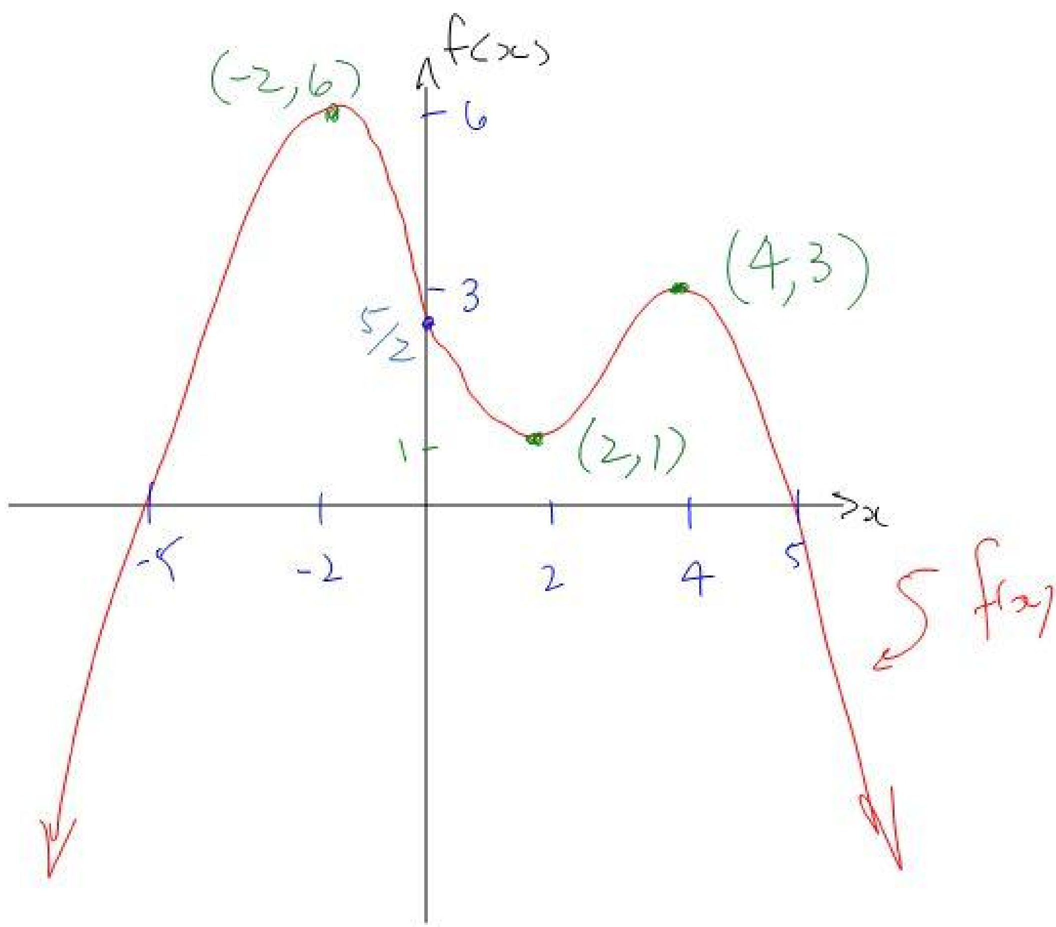
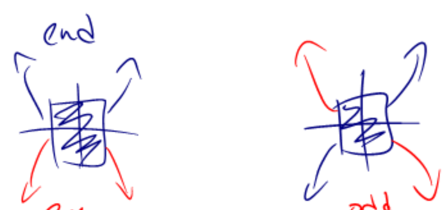


Figure 2.2.1



Observations about $f(x)$:

1) $f(x)$ is a polynomial of even order (degree).

2) The leading coefficient is negative $\because$ as $x \to \pm\infty$, $f(x) \to -\infty$

3) $f(x)$ has 3 turning points (where the functional behaviour of INCREASING/DECREASING switches from one to the other.)

4) $f(x)$ has 2 zeros: $f(-5)=0$, $f(5)=0$

5) $f(x)$ is increasing on $(-\infty,-2)\cup(2,4)$

$f(x)$ is decreasing on $(-2,2)\cup(4,\infty)$

6) $f(x)$ has a MAXIMUM functional value. $f(-2)=6$

6 is the very highest that f(x) 'goes'. We call such max's GLOBAL maxima (or ABSOLUTE max's)

NICK P. FARTED LOUDLY.

no lowest value

7) $f(x)$ has a LOCAL minimum at $(2,1)$ but No GLOBAL min. (Since $f(x)\to-\infty$)

In a neighbourhood of $x=2$ (eg $x\in[0,3]$ – a restricted part of the domain)
$f(2)=1$ is the lowest functional value

Note: at $(4,3)$ we have a local max value of 3 (in a neighbourhood of $x=4$)

Note further: All turning are max/mins. Usually only local, sometimes global.

Consider the sketch of the graph of some function $g(x)$:

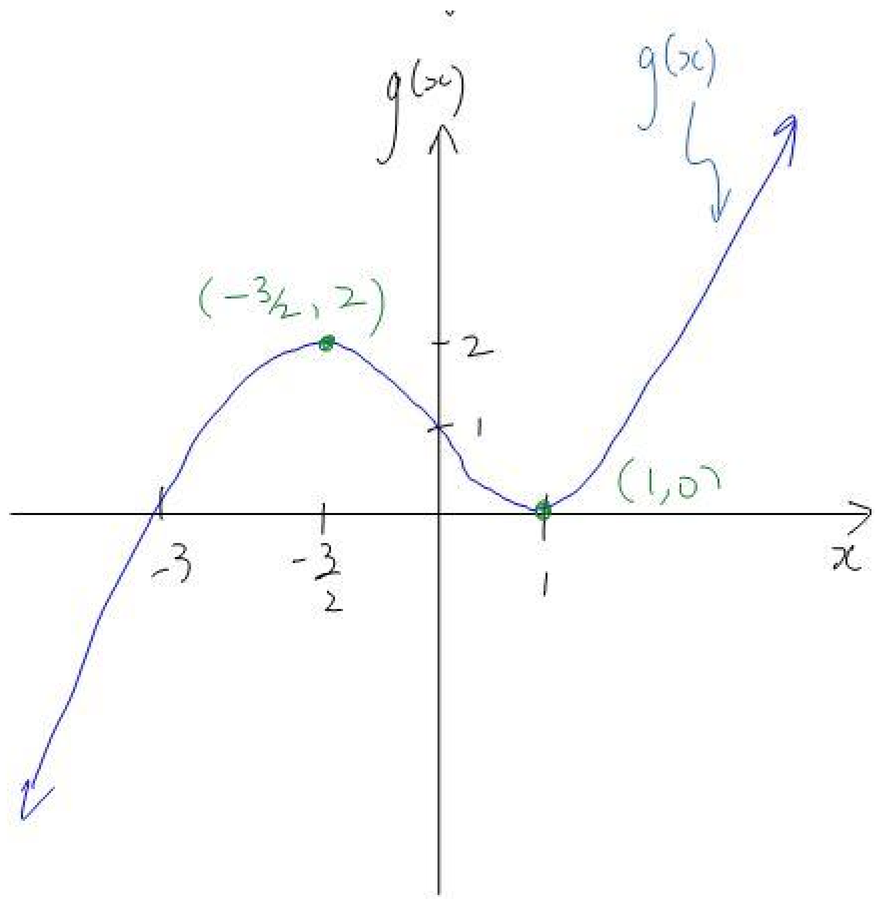


Figure 2.2.2

Observations about $g(x)$:

- $g(x)$ is odd ordered
- the lead coefficient is positive
- $g(x)$ has 2 turning pts
- $g(x)$ is increasing on $(-\infty, -\frac{3}{2}) \cup (1, \infty)$
- $g(x)$ is decreasing on $(-\frac{3}{2}, 1)$
- $g(x)$ has 2 zeros $f(-3) = 0$, $f(1) = 0$ — $x \to \infty$, $f(x) \to \infty$
- $g(x)$ has a local max of 2 at $x = -\frac{3}{2}$ but no global max
- $g(x)$ has a local min of 0 at $x = 1$ but no global min

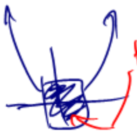
General Observations about the Behaviour of Polynomial Functions

1) The Natural Domain of all Polynomial Functions is $(-\infty, \infty)$ | $x \in \mathbb{R}$

2) The Range of ODD ORDERED Polynomial Functions is $(-\infty, \infty)$ | $f(x) \in \mathbb{R}$

3) The Range of EVEN ORDERED Polynomial Functions depends. (on the sign of lead coefficient)

positive lead coeff



there will be a global min "m"
Then the range is $[m, \infty)$

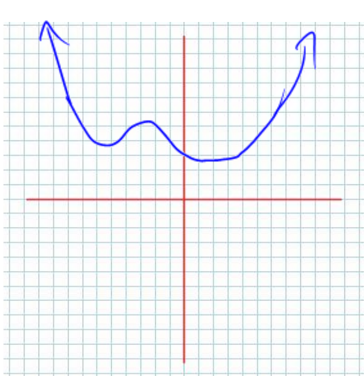
neg lead coeff

global max M
with range $(-\infty, M]$

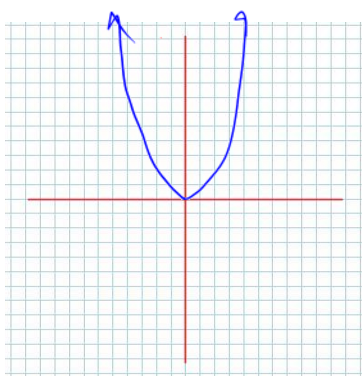
Even Ordered Polynomials

Zeros: A Polynomial Function, $f(x)$, with an even degree of "n" (i.e. $n = 2, 4, 6...$) can have $0, 1, 2, 3, ..., n$

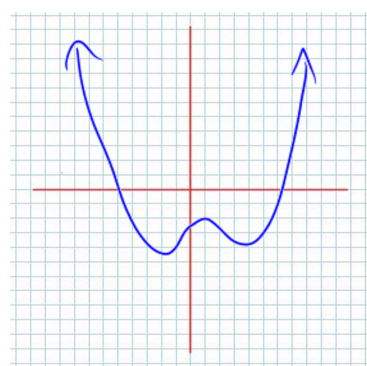
e.g. A degree 4 Polynomial Function (with a positive leading coefficient) can look like:



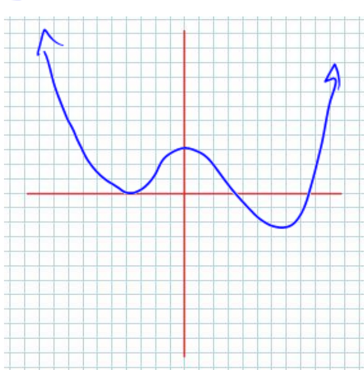
0 zeros



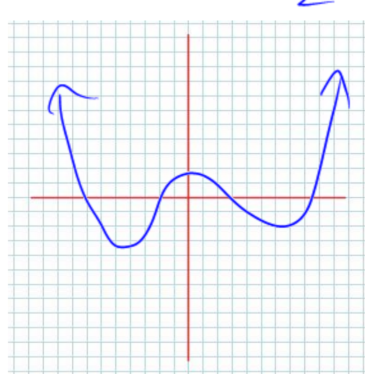
1 zero



2 zeros



3 zeros



4 zeros

Turning Points:

The minimum number of turning points for an Even Ordered Polynomial Function is 1 (the fn has to turn

The maximum number of turning points for a Polynomial Function of (even) order n is $n-1$

Note: even ordered poly. fns. must have an odd # of turning points

Odd Ordered Polynomials

Zeros: min: 1 zero

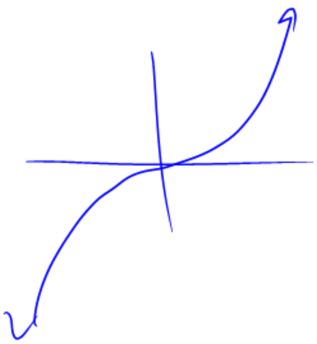
order n (n odd) max: n → n is odd → eg $g(x) = \underbrace{(x-2)(x-1)(x+1)}_{\text{order } 3 - 3 \text{ zero}}$

Turning Points:

order n (n odd)

min: 0

max: $n-1$



Note! odd ordered poly fns must have an even number of turning points

Example 2.2.1 (*#2, for #1b, from Pg. 136*)

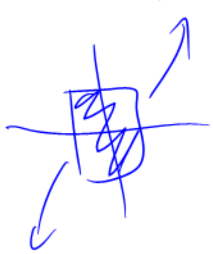
Determine the minimum and maximum number of zeros and turning points the given function may have: $g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$

order: 5

lead coeff: +2

zeros: min: 1

max: 5



Turning pts: min: 0

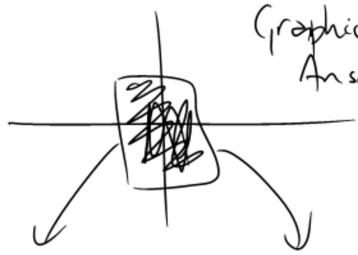
max: 4

Example 2.2.2 (*#4d from Pg. 136*)

Describe the end behaviour of the polynomial function using the order and the sign on the leading coefficient for the given function: $f(x) = -2x^4 + 5x^3 - 2x^2 + 3x - 1$

f(x) is even ordered ⇒ or . f(x) has a neg. lead coefficient

∴ The End Behaviour is



Graphical Answer

Algebraic Answer

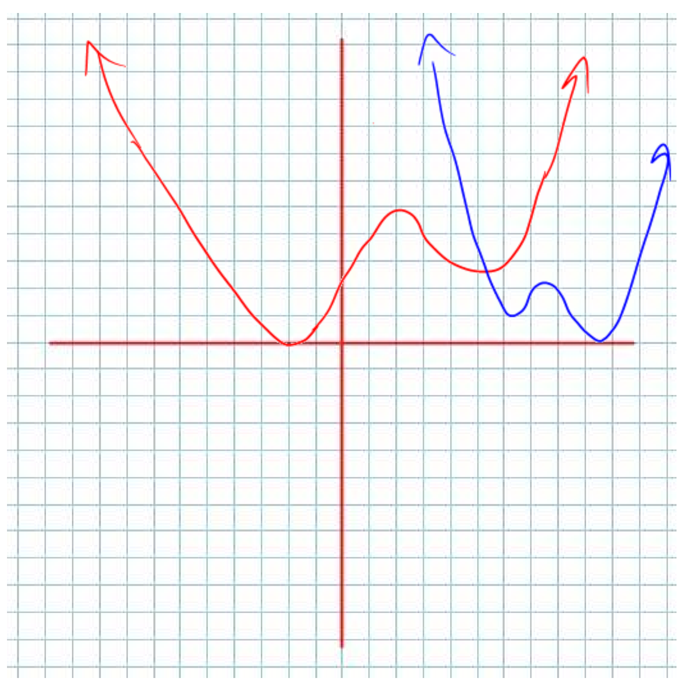
$x \to -\infty$, $f(x) \to -\infty$

$x \to +\infty$, $f(x) \to -\infty$

Example 2.2.3 (*#7c from Pg. 137*)

Sketch a graph of a polynomial function that satisfies the given set of conditions:

Degree 4 - positive leading coefficient - 1 zero - 3 turning points.



> *Class/Homework for Section 2.2*
>
> *Pg. 136 - 138 #1 – 5, 7, 8, 10, 11*