Happy Birthday Olivia

2.3 Zeros of Polynomial Functions

(*Polynomial Functions in Factored Form*)

Today we take a deeper look inside the Box of Mystery, carefully examining Zeros of Polynomial Functions

We'll begin with an **Algebraic Perspective**:

Consider the polynomial function in factored form:

$$f(x) = (2x-3)(x-1)(x+2)(x+3)$$

Observations:

$f(x)$ has 4 distinct zeros

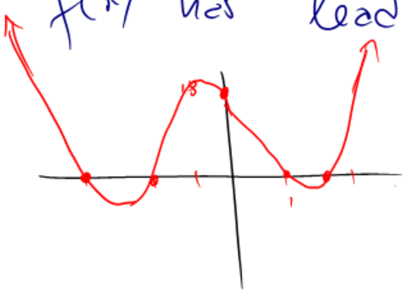
$x = \frac{3}{2}, 1, -2, -3$

'y int' $f(0) = 18$

$(0, 18)$

$f(x)$ has lead term $2x^4$

rough sketch



Note: This sketch is only a possible sketch. It's likely 'wrong'. For more accuracy use graphing tech or Calculus

Now, consider the polynomial function $g(x) = (x-3)^2(x-1)(x+2)$

Observations: $g(x)$ has 3 zeros but one is "repeated"

$x = 3$ order 2 zero

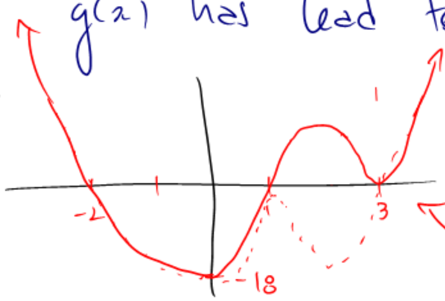
$x = 1$

$x = -2$

y int $g(0) = -18$

$g(x)$ has lead term $1x^4$

Rough Sketch

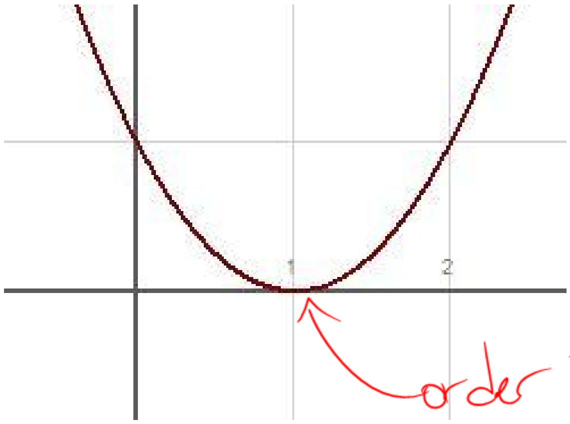


"shape" of an order 2 zero?

Geometric Perspective on Repeated Roots (zeros) of order **2**

Note: order 1 zeros just "cut through" the domain axis (like a line)

Consider the quadratic in factored form: $f(x) = (x-1)^2$



order 2 zeros are turning points

Figure 2.3.1

Consider the polynomial function in factored form: $h(t) = (t+1)^3(2t-5)$

Observations: h(t) has 2 zeros

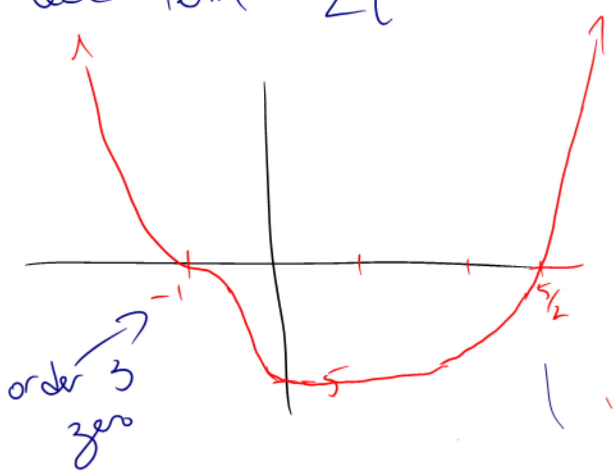
$t = -1$ (order 3 zero)

$t = \frac{5}{2}$

y int
$h(0) = -5$

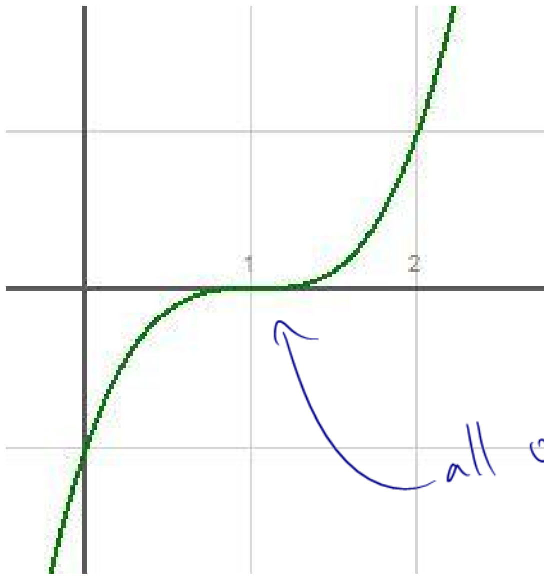
h(t) has lead term $2t^4$

Rough Sketch

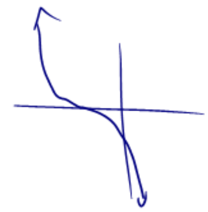


Geometric Perspective on Repeated Roots (zeros) of order **3**

Consider the function $f(x) = (x-1)^3$



or like

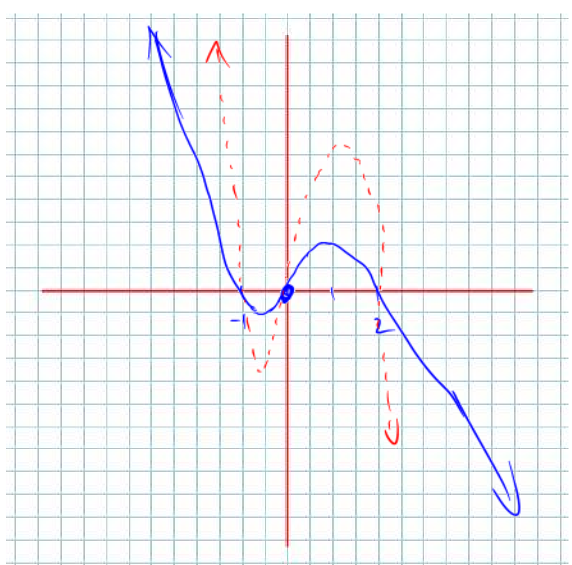


all order 3 zeros look like this (not a turning point!)

Figure 2.3.2

Example 2.3.1

Sketch a (**possible**) graph of $f(x) = -2x(x+1)(x-2)$



f(x) has 3 zeros

$x = 0, -1, 2$

(all order 1)

f(x) has lead term

$-2x^3$

y int $f(0) = 0$

VHA

Families of Functions

Polynomial functions which share the same order are "broadly related" (e.g. **all** quadratics are in the "order 2 family").

Polynomial Functions which share the same order and zeros are more tightly related.

Polynomial Functions which share the same order and zeros and end behaviour are like siblings.

Lead coefficient distinguishes between "family members"

Example 2.3.2

The family of functions of order 4, with zeros $x = -1, 0, 3, 5$ can be expressed as:

$$f(x) = ax(x+1)(x-3)(x-5)$$

lead coefficient

Example 2.3.3

Sketch a graph of $g(x) = 4x^4 - 16x^2$

g(x) is of order 4 with lead coefficient 4

FACTOR TO FIND ZEROS

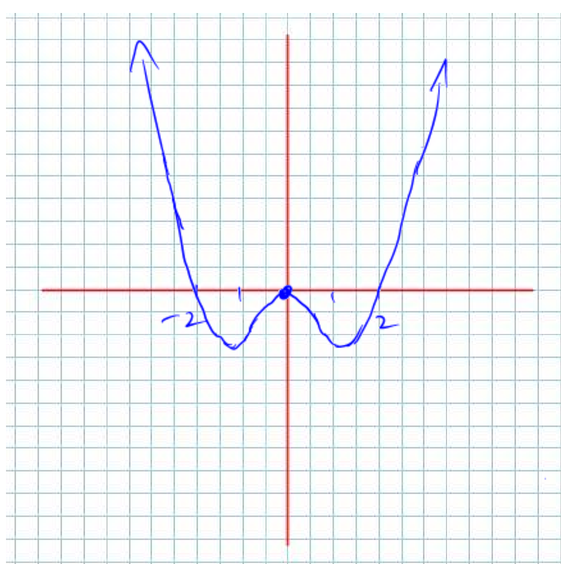
$$g(x) = 4x^2(x^2 - 4)$$

$$= 4x^2(x-2)(x+2)$$

3 zeros

$x = 0$ order 2

$x = 2$
$x = -2$ } order 1



y-int: $g(0) = 0$

y-int

$h(0) = (-1)^3(2)^2 = -4$

Example 2.3.4

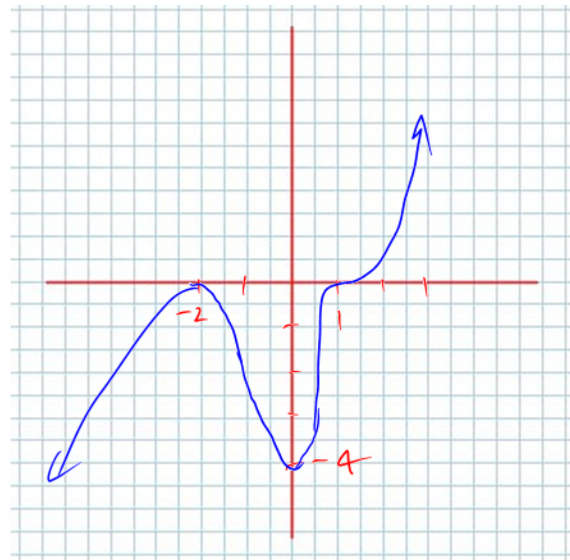
Sketch a (possible) graph of $h(t) = (t-1)^3(t+2)^2$

$h(t)$ has 2 zeros

$t = 1$ (order 3)

$t = -2$ (order 2)

$h(t)$ has lead term t^5



This is a point

Example 2.3.5

Determine **the** quartic function, $f(x)$, with zeros at $\underbrace{x = -2,\ 0,\ 1,\ 3}_{\text{all order 1}}$, if $f(-1) = -2$.

$f(x) = a x(x+2)(x-1)(x-3)$

use $(-1,-2)$ to find a

$-2 = a(-1)(+1)(-2)(-4)$

$\Rightarrow a = \frac{-1}{4}$

$\therefore \boxed{f(x) = \frac{-1}{4} x(x+2)(x-1)(x-3)}$

Class/Homework for Section 2.3

READ ex 3, 4, 5 on Pg 141 - 144

Pg. 146 - 148 #1, 2, 4, 6, 8ab, 10, 12, 13b