2.4a Dividing a Polynomial by a Polynomial

(*The Hunt for Factors*)

Note: In this course we will almost always be dividing a polynomial by a ~~monomial~~ linear factor

eg $(x-2)$ or $(3x-1)$

Before embarking, we should consider some "basic" terms (and notation):

thing being divided

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

stuff left 'over'

thing doing the dividing

answer

form of division statements

× both sides by 'divisor' ⇒

$$\boxed{\text{dividend} = (\text{divisor})(\text{quotient}) + \text{remainder}}$$

Note: The Divisor and the Quotient will both be FACTORS of the dividend

IF the remainder is zero

Example 2.4.1

Use LONG DIVISION for the following division problem:

$$\frac{5x^4+3x^3-2x^2+6x-7}{x-2}$$

Please read Example 1 (Part A) on Pgs. 162 – 163 in your textbook.

turn into a "long division statement"

$$\begin{array}{r|l} & 5x^3+13x^2+24x+54 \\ \hline x-2 & 5x^4+3x^3-2x^2+6x-7 \\ & 5x^4-10x^3 \\ \hline & +13x^3-2x^2 \\ & 13x^3-26x^2 \\ \hline & 24x^2+6x \\ & 24x^2-48x \\ \hline & 54x-7 \\ & 54x-108 \\ \hline & +101 \end{array}$$

$3x^3-(-10x^3)$

remainder

INTO
TIMES
SUBTRACT
BRING DOWN

write the division statement

$\therefore 5x^4+3x^3-2x^2+6x-7=(x-2)(5x^3+13x^2+24x+54)+101$

KEY OBSERVATION:

Since the remainder (R) is not zero

Therefore $(x-2)$ is NOT a factor of the polynomial

Example 2.4.2

Using Long Division, divide $\frac{2x^5+3x^3-4x-1}{x-1}$.

$$
\begin{array}{r}
2x^4 + 2x^3 + 5x^2 + 5x + 1 \\
x-1 \overline{\big) 2x^5 + 0x^4 + 3x^3 + 0x^2 - 4x - 1} \\
\underline{2x^5 - 2x^4} \\
2x^4 + 3x^3 \\
\underline{2x^4 - 2x^3} \\
5x^3 + 0x^2 \\
\underline{5x^3 - 5x^2} \\
5x^2 - 4x \\
\underline{5x^2 - 5x} \\
x - 1 \\
\underline{x - 1} \\
0
\end{array}
$$

(every 'power' needs to show up)

remainder is zero

Division Statement

$\therefore 2x^5+3x^3-4x-1 = (x-1)(2x^4+2x^3+5x^2+5x+1)$

KEY OBSERVATION: $x-1$ is a factor of $2x^5+3x^3-4x-1$ because remainder is 0

($\therefore x=1$ is a zero!)

Classwork: Pg. 169 #5 **(Yep, that's it for today)**