2.4b Dividing a Polynomial by a Polynomial

(The Hunt for Factors – Part 2)

Here we will examine an alternative form of polynomial division called **Synthetic Division**. Don't be fooled! This is not "fake division". You're thinking with the wrong meaning for "synthetic". (Do a search online and see if you can come up with the meaning I am taking!)

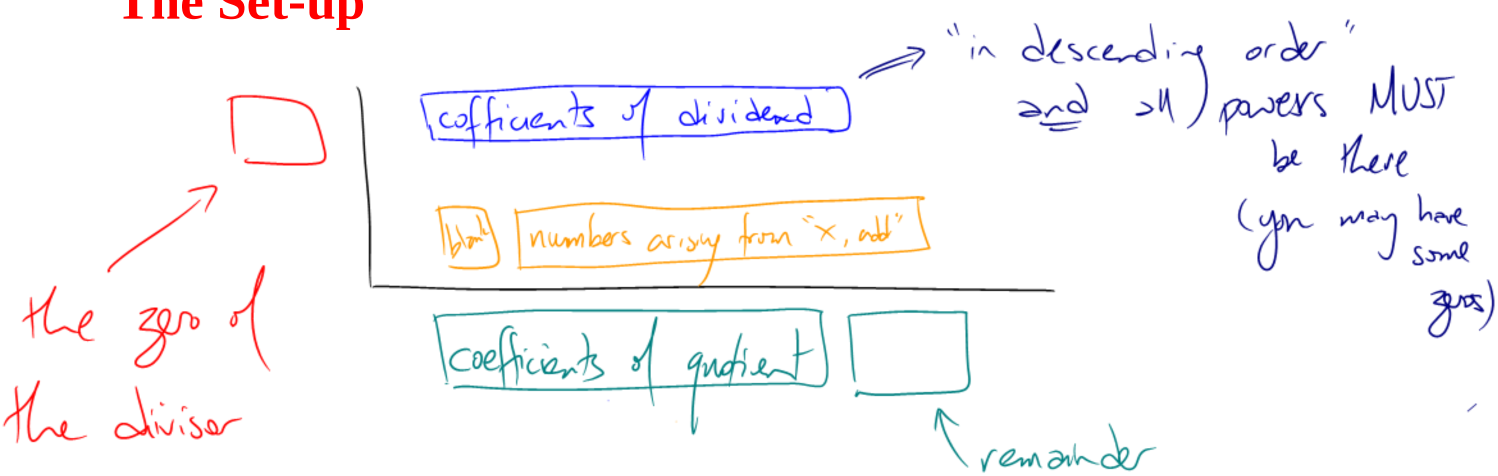
In Synthetic Division we concern ourselves with coefficients of the dividend and the zero of the LINEAR divisor. → $x-1$, $2x+3$

Synthetic Division uses numbers – No variables! (order 1)

w/ operations "BRING DOWN, TIMES, ADD"

> **Note:** Synthetic Division only works with linear divisors. For any non-linear divisors, long division must be use — order is 2 or greater

The Set-up



Example 2.4.3

Divide using synthetic division:

$$(4x^3-5x^2+2x-1)\div(x-2)$$

zero of this is 2

BRING DOWN

TIMES

ADD

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & x^0 \\ 2 & 4 & -5 & 2 & -1 \\ & \downarrow & 8 & 6 & 16 \\ \hline & 4 & 3 & 8 & 15 \\ & x^2 & x & x^0 & \text{remainder} \end{array}$$

$$\therefore\ 4x^3-5x^2+2x-1 = (x-2)(4x^2+3x+8)+15$$

dividend = (divisor)(quotient) + remainder.

Example 2.4.4

Divide using synthetic division:

$$\frac{4x^4+3x^2-2x+1}{x+1}$$

$x+1=0$

$\Rightarrow x=-1$

$$\begin{array}{r|rrrrr} -1 & 4 & 0 & 3 & -2 & 1 \\ & \downarrow & -4 & 4 & -7 & 9 \\ \hline & 4 & -4 & 7 & -9 & 10 \end{array}$$

$$\therefore\ 4x^4+3x^2-2x+1 = (x+1)(4x^3-4x^2+7x-9)+10$$

Example 2.4.5

Divide using your choice of method (and you choose synthetic division…amen)

$$(2x^3-9x^2+x+12)\div(2x-3)$$

zero of divisor $x=\frac{3}{2}$

$2x-3=0$
$2x=3$
$x=3/2$

fractional zeros introduce weirdness.

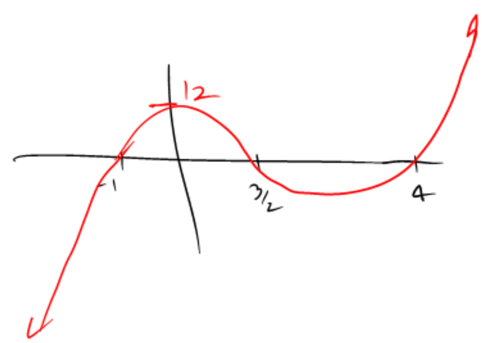
$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & -9 & 1 & 12 \\ & \downarrow & 3 & -9 & -12 \\ \hline & 2 & -6 & -8 & 0 \\ \div 2 & 1 & -3 & -4 & \end{array}$$

÷ quotient coefficients by the denom. of the zero

LEAVE THE REMAINDER ALONE

$$\Rightarrow 2x^3-9x^2+x+12=(2x-3)(x^2-3x-4)$$
$$=(2x-3)(x-4)(x+1)$$

We could sketch!



Example 2.4.6

Is $3x-1$ a factor of the function $f(x)=6x-x^3+2+3x^4$?

⇒ dividing will tell us

$$\begin{array}{r|rrrrr} & x^4 & x^3 & x^2 & x^1 & x^0 \\ \frac{1}{3} & 3 & -1 & 0 & 6 & 2 \\ & \downarrow & 1 & 0 & 0 & 2 \\ \hline & 3 & 0 & 0 & 6 & 4 \\ \div 3 & 1 & 0 & 0 & 2 & \end{array}$$

Answer the question asked the division statement isn't need.

Ans: Since the remainder is not zero $(r=4)$
$\therefore$ $3x-1$ is NOT a factor of $f(x)$

Example 2.4.7 (OK…this is a lot of examples!)

Consider again (from **Example 2.4.6**) $f(x)=3x^4-x^3+6x+2$, and calculate $f\left(\frac{1}{3}\right)$. zero of divisor from ex. 6.

$$f\left(\tfrac{1}{3}\right)=3\left(\tfrac{1}{3}\right)^4-\left(\tfrac{1}{3}\right)^3+6\left(\tfrac{1}{3}\right)+2$$

$$=4$$

WHOA! That was the remainder of the division!

Example 2.4.8

Consider **Example 2.4.5**. Let $g(x)=2x^3-9x^2+x+12$, and calculate $g\left(\frac{3}{2}\right)$.

$$g\left(\tfrac{3}{2}\right)=2\left(\tfrac{3}{2}\right)^3-9\left(\tfrac{3}{2}\right)^2+\left(\tfrac{3}{2}\right)+12$$

$$=2\left(\tfrac{27}{8}\right)-9\left(\tfrac{9}{4}\right)+\tfrac{3}{2}+12$$

$$=\tfrac{27}{4}-\tfrac{81}{4}+\tfrac{6}{4}+\tfrac{48}{4}=0$$

The Remainder Theorem

Given a polynomial function, $f(x)$, divided by a linear binomial, $x-k$, then the remainder of the division is the value $f(k)$ $\Rightarrow$ calculated!

(no division necessary!)

Proof of the Remainder Theorem

Consider the division $f(x) \div (x-k)$

Division Statement

the quotient fn

$$f(x) = (x-k)\,q(x) + r$$

$$\therefore \quad f(k) = (\cancel{k-k})^{0} \cdot q(k) + r$$

$$= r$$

$\square$

Example 2.4.9

Determine the remainder of $\dfrac{5x^4 - 3x^3 - 50}{x-2}$.

zero is 2.

WAIT!!!! We MUST have a FUNCTION

So create one.

Let $f(x) = 5x^4 - 3x^3 - 50$

then the remainder is

$$f(2) = 5(2)^4 - 3(2)^3 - 50$$

$$= 6$$

Class/Homework for Section 2.4

Pg. 168 - 170 #2, 6acdef, 10acef, 12, 13