2.5 The Factor Theorem

(Factors have been FOUND)

The Factor Theorem

Given a polynomial function, $f(x)$, then $x-a$ is a factor of $f(x)$ IF AND ONLY IF ($\Longleftrightarrow$)

$f(a) = 0$

Example 2.5.1

Use the Factor Theorem to factor $x^3 + 2x^2 - 5x - 6$.

WAIT!!!! We MUST have a FUNCTION (holy smokes)

Let $f(x) = x^3 + 2x^2 - 5x - 6$

Test values (factors of "6")

$\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = 0$

$\Rightarrow (x+1)$ is a factor

Use Synth. Division

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$\therefore x^3 + 2x^2 - 5x - 6 = (x+1)\underbrace{(x^2 + x - 6)}$ does this factor? Yes!

$= (x+1)(x+3)(x-2)$

FACTOR FULLY!!

Consider $f(x)$ in factored form

$f(x) = (x-a)(x-b)(x-c)$ where a,b,c are zeros of $f(x)$ IF they exist

Notice $|(a)(b)(c)| = 6$

$\Rightarrow$ the zeros are factors of the constant term

Example 2.5.2

Factor **fully** $x^4 - x^3 - 16x^2 + 4x + 48$

TEST VALUES (factors of 48)

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$

Let $f(x) = x^4 - x^3 - 16x^2 + 4x + 48$

$f(2) = 0$ (As soon as you find the zero, write it down)

$(\therefore x - 2$ is a factor$)$

Synthetic Div

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & -16 & 4 & 48 \\ & \downarrow & 2 & 2 & -28 & -48 \\ \hline & 1 & 1 & -14 & -24 & 0 \end{array}$$

apply the factor theorem to cubic

$\therefore f(x) = (x-2)(x^3 + x^2 - 14x - 24)$

Let $g(x) = x^3 + x^2 - 14x - 24$

no need to test

TEST VALUES

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$g(-2) = 0$ $((x+2)$ is a factor$)$

Synth Div

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$\therefore x^4 - x^3 - 16x^2 + 4x + 48 = (x-2)(x+2)(x^2 - x - 12)$

$= (x-2)(x+2)(x+3)(x-4)$

Example 2.5.3 (*Pg 177 #6c in your text***)**

Factor fully $x^4+8x^3+4x^2-48x$

Common factor first if you can.

Let $f(x) = x^4 + 8x^3 + 4x^2 - 48x$

$= x(x^3 + 8x^2 + 4x - 48)$

Test Values

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm16, \pm24, \pm48$

$f(2) = 0$ ($\Rightarrow$ $(x-2)$ is a factor!)

Synth Div

$$\begin{array}{r|rrrr} 2 & 1 & 8 & 4 & -48 \\ & & 2 & 20 & 48 \\ \hline & 1 & 10 & 24 & 0 \end{array}$$

$\therefore x^4 + 8x^3 + 4x^2 - 48x = x(x-2)(x^2+10x+24)$

$= x(x-2)(x+4)(x+6)$

Example 2.5.4 (*Pg 177 #10*)

1ST bit of info.

2ND bit of information

When ax^3-x^2+2x+b is divided by $x-1$ the remainder is 10. When it is divided by $x-2$ the remainder is 51. Find a and b.

This problem is very instructive.

It brings tears of joy to my eyes.

Let $f(x) = ax^3 - x^2 + 2x + b$

$f(1) = 10$

$\Rightarrow a(1)^3 - (1)^2 + 2(1) + b = 10$

$a + b = 9$ ①

$f(2) = 51$

$\Rightarrow a(2)^3 - (2)^2 + 2(2) + b = 51$

$8a + b = 51$ ②

use elimination or substitution to solve the S.O.L.E

System of Linear Equations

② − ①

elimination

$7a = 42$

$\Rightarrow \boxed{a=6}$ $\quad \therefore \boxed{b=3}$ (by ①)

Class/Homework for Section 2.5

Pg. 176 - 177 #1, 2, 5 – 7 abcd, 8ac, 9, 12 (angels sing over 9 & 12)