A∞Ω (Period 4)
MATH@TD

SOLUTIONS

MHF4U - Practice Quiz on 2.5 2.6

1. Which one of the following functions is divisible by $x - 3$? ⟸ zero of divisor is 3.

   a. $f(x) = x^3 - 10x^2 + 29x - 24$
   b. $g(x) = x^3 - 4x^2 - 13x + 24$
   c. $h(x) = 3x^3 - 6x^2 + 3x + 9$
   d. $j(x) = x^3 - 9x^2 + 15x - 12$

   using the Remainder theorem

   $f(3) = (3)^3 - 10(3)^2 + 29(3) - 24$
   $= 0$

   means "no remainder"

   ∵ the remainder is zero ⟹ $f(x)$ is divisible by $x - 3$

2. Which one of the following functions is divisible by $x + 1$? ⟸ zero $x = -1$

   a. $f(x) = x^3 - 2x^2 + x - 3$
   b. $g(x) = x^2 - 3x + 1$
   c. $h(x) = x^5 - x^4 - 2x^3 + 8x^2 - 6x$
   d. $j(x) = x^5 + x^4 - 2x^3 + 4x^2 + 6x$

   $f(-1) = -7$ No
   $g(-1) = -1$ No
   $h(-1) = 2$ No
   $j(-1) = 0$ yes.

3. Which one of the following is not a factor of $f(x) = -6x^3 - 47x^2 - 97x - 60$?

   a. $-2x - 3$ zero $-3/2$
   b. $3x + 4$ zero $-4/3$
   c. $3x - 4$ zero $4/3$
   d. $x + 5$ zero -5

   $f(-3/2) = 0$ ∴ factor
   $f(-4/3) = 0$ ∴ factor
   $f(4/3) = -28\,4/9 \neq 0$ ∴ not a factor

4. What is the remainder when $x - 4$ is divided into $3x^4 - 9x^3 + 6x^2 - 8x + 11$?

   a. 1483
   b. 267
   c. 11
   d. −245

   zero of divisor $x = 4$.

   LET $f(x) = 3x^4 - 9x^3 + 6x^2 - 8x + 11$ (remainder theorem)

   $f(4) = 3(4)^4 - 9(4)^3 + 6(4)^2 - 8(4) + 11$
   $= 267$

If you use the Remainder Theorem or the Factor Theorem, create a fn (if you don't have one)!

5. The polynomial $-6x^3 - 55x^2 + kx - 16$ has factors $x + 8$ and $3x + 2$. Determine the value of k.

a. -58
b. -16
c. 16 zero -8 zero $-\frac{2}{3}$
d. 58

use either one
(since there is only one unknown
$\Rightarrow$ we only need one bit of info)

Let $f(x) = -6x^3 - 55x^2 + kx - 16$

using -8

$f(-8) = -6(-8)^3 - 55(-8)^2 - 8k - 16$

$= -464 - 8k$

(Note: $f(-\frac{2}{3})$ gives the same result)

$\because$ $x+8$ is a factor $\Rightarrow f(-8) = 0$

$\Rightarrow -464 - 8k = 0$

$\Rightarrow k = -58$

" $-$ $+$ $+$ " (order of signs)

6. Write the expression $27x^3 - 1$ in factored form.

$3x$ 1 (cube roots)

a. $(3x + 1)(3x - 1)$
b. $(9x - 1)(81x^2 + 9x + 1)$
c. $(3x + 1)(9x^2 - 3x + 1)$
d. $(3x - 1)(9x^2 + 3x + 1)$

7. Write the expression $x^9y^3 + 216z^3$ in factored form.

a. $(x^3y + 6z)(x^6y^2 - 6x^3yz + 36z^2)$
b. $(x^3y - 6z)(x^6y^2 + 6x^3yz + 36z^2)$
c. $(x^3y + 6z)(x^6y^2 + 6x^3yz + 36z^2)$
d. $(x^3y + 6z)(x^3y - 6z)$

$(x^3y + 6z)((x^3y)^2 - (x^3y)(6z) + (6z)^2)$

8. The factor theorem says that $x - a$ is a factor of $f(x)$ if and only if what is true?

$f(a) = 0$ Amen and amen.

9. The remainder theorem says that when a polynomial, $f(x)$, is divided by $x - a$, the remainder is equal to what?

$f(a)$ is the remainder.

10. State the remainder when $x - 5$ is divided into the polynomial $3x^3 - 8x^2 + 6x - 10$.

Let $f(x) = 3x^3 - 8x^2 + 6x - 10$

(I'm using the remainder theorem, but you could also divide)

$f(5) = 3(5)^3 - 8(5)^2 + 6(5) - 10$

$= 195$

$\therefore$ The remainder is 195

11. Factor the polynomial $-2x^3+12x^2+8x-48$ using the factor theorem.

Let $f(x) = -2x^3+12x^2+8x-48$ (common factor if you can)

TEST VALUES

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$\Rightarrow f(x) = -2(x^3-6x^2-4x+24)$

$f(2)=0 \Rightarrow (x-2)$ is a factor

2	1	-6	-4	24
		2	-8	-24
	1	-4	-12	0

don't forget the common factor!

$\therefore f(x) = -2(x-2)(x^2-4x-12)$

$= -2(x-2)(x-6)(x+2)$

12. Factor the polynomial $x^3+6x^2-19x-24$ using the factor theorem.

Let $f(x) = x^3+6x^2-19x-24$

TEST VALUES

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm24$

$f(3)=0 \Rightarrow (x-3)$ is a factor

3	1	6	-19	-24
		3	27	24
	1	9	8	0

$\therefore x^3+6x^2-19x^2-24 = (x-3)(x^2+9x+8)$

$= (x-3)(x+1)(x+8)$

13. Sketch $f(x) = x^4-2x^3-13x^2+38x-24$ by finding all zeros of $f(x)$.

YES! Best Question EVER!!!

$f(1)=0$

TEST VALUES

$\pm1, \pm2, \pm3, \pm4$

$\pm6, \pm12, \pm24$

1	1	-2	-13	38	-24
		1	-1	-14	24
	1	-1	-14	24	0

$\therefore f(x) = (x-1)(x^3-x^2-14x+24)$

Let $g(x) = x^3-x^2-14x+24$ " "

$g(2)=0$

2	1	-1	-14	24
		2	2	-24
	1	1	-12	0

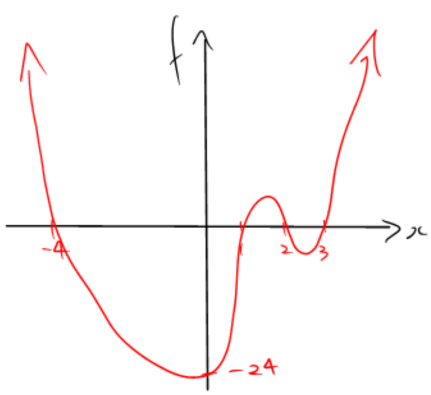
$\therefore f(x) = (x-1)(x-2)(x^2+x-12)$

$= (x-1)(x-2)(x-3)(x+4)$

zeros

$x = 1, 2, 3, -4$

y-int $f(0) = -24$



(basic sketch)

14. The polynomial $-6x^3+50x^2+ax+24$ has factors $x-1$ and $2x+1$. Determine the value of a.

Let $f(x) = -6x^3 + 50x^2 + ax + 24$

By the Factor Theorem

$f(1)=0$ (or $f(-\frac{1}{2})=0$)

$\Rightarrow -6(1)^3 + 50(1)^2 + a + 24 = 0$

$\Rightarrow a = -68$

$x-1$ and $2x+1$ are NOT both factors!

Note BAD QUESTION!

$f(-\frac{1}{2}) = 0$

$\Rightarrow -6(\frac{-1}{2})^3 + 50(-\frac{1}{2})^2 - \frac{1}{2}a + 24 = 0$

$\Rightarrow \frac{6}{8} + \frac{50}{4} - \frac{1}{2}a + 24 = 0$

$\Rightarrow a \doteq \underline{74.5} \neq -68$

15. Factor $x^3y^6+8z^3$.

$= (xy^2 + 2z)(x^2y^4 - 2xy^2z + 4z^2)$

16. Factor $\frac{1}{64}x^3 - \frac{8}{27}$.

$\left(\frac{1}{4}x - \frac{2}{3}\right)\left(\frac{1}{16}x^2 + \frac{1}{6}x + \frac{4}{9}\right)$

Happy Studying!