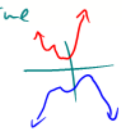
Chapter 2: Practice Test — Solutions

1. Which of the following statements about a polynomial function is false?
   a. A polynomial function of degree n has at most n turning points. incorrect. '$n-1$' turning pts.
   b. A polynomial function of degree n may have up to n distinct zeros. true → think FACTORS
   c. A polynomial function of odd degree must have at least one zero. true

fn must cross domain axis

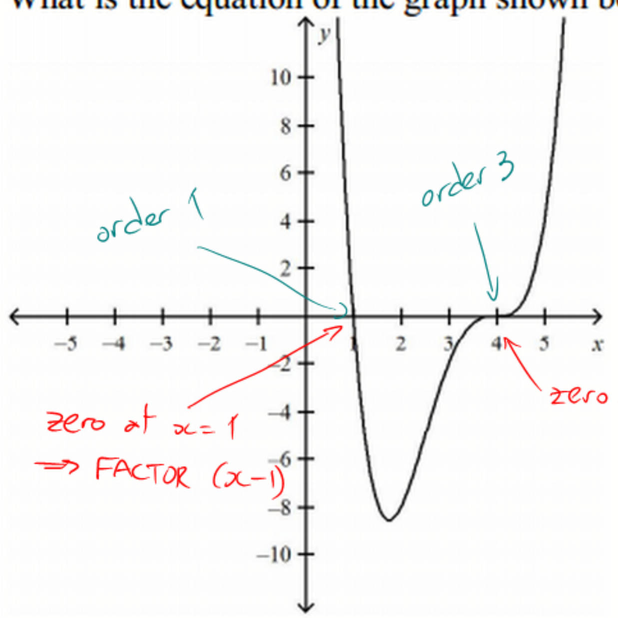
   d. A polynomial function of even degree may have no zeros. true



global min may be above domain axis, or global min may be below domain axis and never cross it.

2. What is the degree and lead coefficient of $f(x) = -x + 5x^2 - 6x^3 + 10$?
   a. degree 1 with a lead coefficient of –1
   b. degree 3 with a lead coefficient of –1
   c. degree 3 with a lead coefficient of –6
   d. degree 6 with a lead coefficient of –1

3. What is the equation of the graph shown below?



zero at $x=4$ ⇒ FACTOR $(x-4)^3$

   a. $f(x) = (x-4)(x-1)$
   b. $f(x) = (x-4)^2(x-1)$
   c. $f(x) = (x+4)^3(x+1)$
   d. $f(x) = (x-4)^3(x-1)$

horizontal dilation $\times \frac{4}{3}$ horizontal translation $\leftarrow 3$

4. Describe the transformations that were applied to $y = x^3$ to create $y = (\frac{3}{4}(x+3))^3 - 2$.

vertical translation $\downarrow 2$

a. horizontally stretched by a factor of $\frac{4}{3}$, horizontally translated 3 units to the left, and vertically translated 2 units down
b. horizontally stretched by a factor of $\frac{3}{4}$, horizontally translated 3 units to the left, and vertically translated 2 units down
c. horizontally stretched by a factor of 3, horizontally translated $\frac{4}{3}$ units to the left, and vertically translated 2 units down
d. horizontally stretched by a factor of $\frac{3}{4}$, horizontally translated 2 units to the right, and vertically translated 3 units up

5. Which expression is the sum of two cubes?

a. $-8x^3 + 64$	c. $9x^3 + 16$
b. $-8x^2 - 64$	d. $25 - 49y^2$

6. Write the expression $27x^3 - 1$ in factored form.

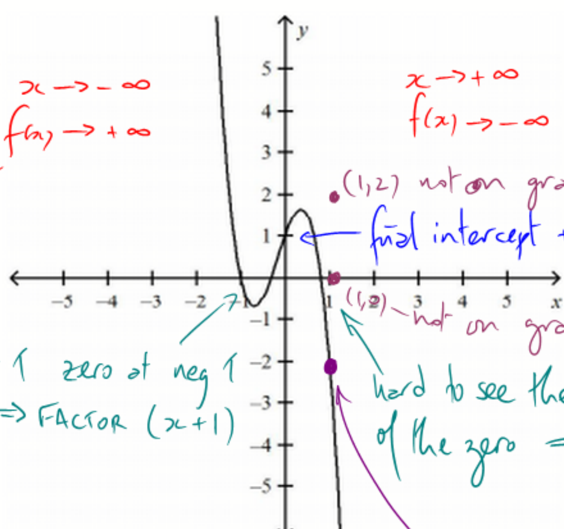
$- \quad + \quad +$

$3x \quad 1$

a. $(3x+1)(3x-1)$	c. $(3x+1)(9x^2 - 3x + 1)$
b. $(9x-1)(81x^2 + 9x + 1)$	d. $(3x-1)(9x^2 + 3x + 1)$

$- \quad (3x)^2 + (3x)(1) + (1)^2$

7. Using end behaviours, and zeros, determine the polynomial equation that represents the graph shown below.



$\begin{cases} x \to -\infty \\ f(x) \to +\infty \end{cases}$ $\quad$ $\left.\begin{matrix} x \to +\infty \\ f(x) \to -\infty \end{matrix}\right\}$ $\Rightarrow$ f(x) is odd ordered w/ neg lead coefficient eg $-4x^3$

(1,2) not on graph

final intercept +1

(1,0) not on graph

order 1 zero at neg 1 $\Rightarrow$ FACTOR $(x+1)$

hard to see the correct value of the zero $\Rightarrow$ I'll try another technique

(1,−2) on graph

using the reasoning outside of this box, the answer is a), c) or d).
Find a point on g(x) and compare w/ the sketch

a. $g(x) = -4x^3 + 3x + 1$	c. $g(x) = -4x^3 + 2x^2 + 3x + 1$
b. $g(x) = -4x^4 - 3x^3 - 2x^2 + 3x + 1$	d. $g(x) = -4x^3 - 2x^2 + 3x + 1$

OR USE THE ZERO $x = -1$

a) $g(1) = 0$, c) $g(1) = +2$ $\quad$ d) $g(1) = -2$ (remember $g(1) = -2$ means $(1,-2)$ belongs to the graph of $g(x)$)

8. If any of the linear factors of a polynomial function are squared, then which of the following is not true of the corresponding x-intercepts? order 2 eg $(x-2)^2$

a. The x-intercepts are turning points of the curve. true

b. The x-axis is tangent to the curve at these points. true

c. The graph passes through the x-axis at these points. incorrect

d. The graph has a parabolic shape near these x-intercepts. true

9. Which one of the following is not a factor of $f(x) = 2x^3 + 9x^2 + 3x - 4$?

a. $2x - 1$ $f(\frac{1}{2}) = 0$ ∴ factor

b. $x - 1$ $f(1) = 10$ ∴ not a factor

c. $x + 4$

d. $x + 1$

check the other two if you wish

10. What is the remainder when $x + 6$ is divided into $x^4 - 2x^3 + x^2 + 4x - 25$?

a. 1715

b. 899

c. –25

d. –949

↳ zero of divisor is -6

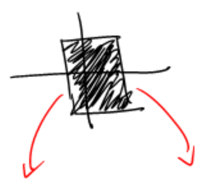
Let $f(x) = x^4 - 2x^3 + x^2 + 4x - 25$: $f(-6) = 1715$

11. Without expanding, state the order (degree), the leading coefficient, and the end behaviours of the polynomial function $g(x) = x(3x - 4)(-2x + 1)(x - 5)$. (you may use a "sketch" to describe the end behaviours).

lead term is $x(3x)(-2x)(x) = -6x^4$ ∴ order 4 w/ lead coefficient -6

end behaviour: algebraic view $\left\{ \begin{matrix} x \to -\infty & x \to +\infty \\ f(x) \to -\infty, & f(x) \to -\infty \end{matrix} \right\}$

geometric view

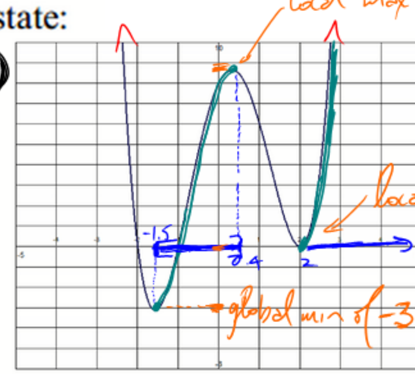


12. Given the sketch of the graph of a polynomial function $f(x)$, state:

a) whether the function is even or odd ordered (with a reason)

b) where the function is increasing

c) any maximums and/or minimums.



local max of 8.6 at $x = 0.4$

local min of 0 at $x = 2$

global min of -3.1 at $x = -1.5$

a) $f(x)$ is even ordered since the two 'sides' of end behaviour are the same (as $x \to \pm\infty$, $f(x) \to +\infty$)

b) $f(x)$ is increasing on (the domain intervals) $(-1.5, 0.4) \cup (2, \infty)$

c) see the sketch for my soln in orange.

x^4 $(x+4)(x-2)^3$ final intercept

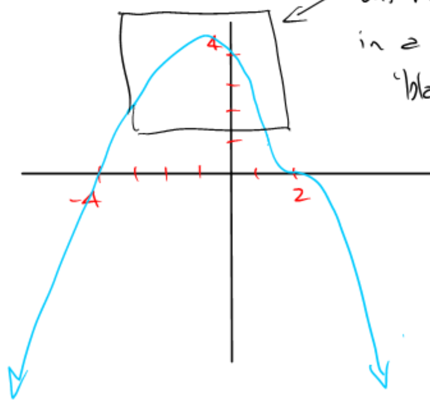
13. Write the equation and sketch an example of a quartic function with the zeros at –4, 2 (order 3), if $f(0) = 4$. What further information about the polynomial function is needed for an "accurate" sketch of the function's graph?

$f(x) = a(x+4)(x-2)^3$ using $(0,4)$, find a.

$4 = a(4)(-2)^3$

$a = -\frac{1}{8}$

$\therefore f(x) = -\frac{1}{8}(x+4)(x-2)^3$



this region is still in a kind of 'black box'

For greater accuracy we need the location of all possible turning points

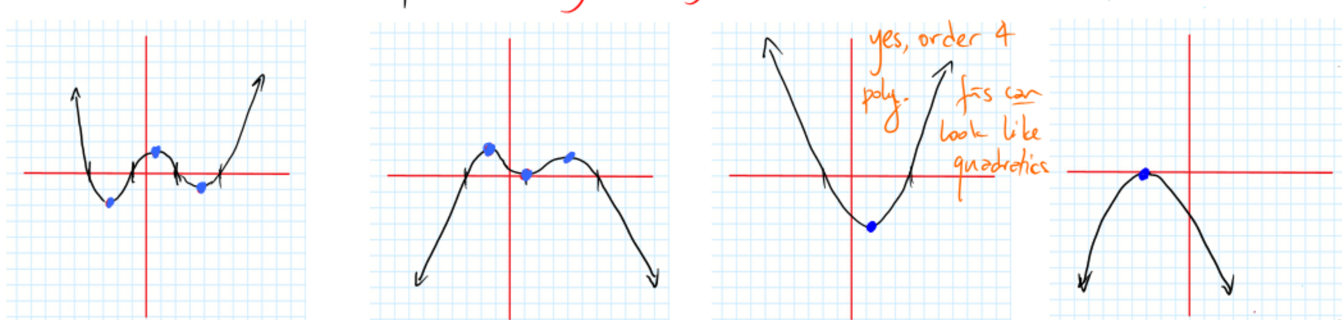
14. Determine the maximum and minimum number of turning points for the polynomial function $g(x) = 4x^4 - 5x^5 + 2x^2 - 7$. Give a reason for your answer.

If this were a test question, your answer would not need to be as detailed as mine.

There is a connection between zeros and turning points.

$g(x)$ is order 4, and because of Factored Form, $g(x)$ can have at most 4 linear factors. So by the FACTOR THEOREM $g(x)$ can have at most 4 zeros. $g(x)$ could otherwise have 3, 2, 1 or 0 zeros.

Consider the sketches of some order 4 polynomial fns with different numbers of zeros. Only necessary turning points will be shown.



Note: a poly. fn. with 0 zeros may also have 3 or 1 turning pts. (imagine shifts, up or down, so the 4 sketches have 0 zeros)

4 zeros requires 3 turning pts

3 zeros requires 3 turning points

2 zeros requires 1 turning point

1 zero requires 1 turning point

The end behaviour of an order 4 polynomial fn requires at least one turning point

The possible functional shapes for an order 4 poly. fn with 4 or 3 zeros requires at most 3 turning pts.

15. State the Remainder Theorem.

Given a fn, $f(x)$, divided by a linear binomial, $(x-k)$, then the remainder of the division is the value $f(k)$

16. Divide: $(x^2 - 6x^4 + 9) \div (x^2 + 2)$. (long division required)

$$
\begin{array}{r|l}
 & -6x^2 + 13 \\
x^2+2 & -6x^4 + 0x^3 + x^2 + 0x + 9 \\
 & -6x^4 \qquad\quad -12x^2 \\
\hline
 & \qquad\qquad 13x^2 + 9 \\
 & \qquad\qquad 13x^2 + 26 \\
\hline
 & \qquad\qquad\quad -17
\end{array}
$$

$$\therefore -6x^4 + x^2 + 9 = (x^2+2)(-6x^3+13) - 17$$

17. Divide $(6x^4 - 6x^3 + 5x^2 - 12x + 1) \div (x + 2)$ using synthetic division.

zero $x = -2$

$$
\begin{array}{r|rrrrr}
-2 & 6 & -6 & 5 & -12 & 1 \\
 & & -12 & 36 & -82 & 188 \\
\hline
 & 6 & -18 & 41 & -94 & 189
\end{array}
$$

$$\therefore 6x^4 - 6x^3 + 5x^2 - 12x + 1 = (x+2)(6x^3 - 18x^2 + 41x - 94) + 189$$

18. When $ax^3 - x^2 + 3x + b$ is divided by $x - 2$, the remainder is 59. When it is divided by $x + 1$, the remainder is -1. Find the values of a and b.

Let $f(x) = ax^3 - x^2 + 3x + b$

① $f(2) = 59$

$\Rightarrow 8a - 4 + 6 + b = 59$

$\Rightarrow 8a + b = 57$

② $f(-1) = -1$

$\Rightarrow -a - 1 - 3 + b = -1$

$\Rightarrow -a + b = 3$

① − ②

$9a = 54$

$\Rightarrow \boxed{a = 6}$

sub into ②

$\Rightarrow \boxed{b = 9}$

19. Sketch $f(x) = 2x^4 + 7x^3 + 3x^2 - 8x - 4$, providing as much deatail about the behaviour of the function as possible. Label at least 4 points.

end behaviour
$x \to \pm\infty, f(x) \to +\infty$

find intercept
$f(0) = -4$

zeros. (TEST VALUES)
$\pm 1, \pm 2, \pm 4$

$f(1) = 0 \Rightarrow (x-1)$ is a factor

Synth Div.

$$\begin{array}{r|rrrrr} 1 & 2 & 7 & 3 & -8 & -4 \\ & & 2 & 9 & 12 & 4 \\ \hline & 2 & 9 & 12 & 4 & 0 \end{array}$$

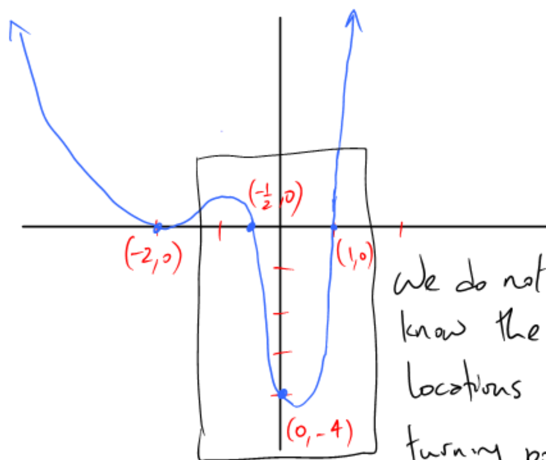
$\therefore f(x) = (x-1)(2x^3 + 9x^2 + 12x + 4)$

Let $g(x) = 2x^3 + 9x^2 + 12x + 4$
$g(-2) = 0$

$$\begin{array}{r|rrrr} -2 & 2 & 9 & 12 & 4 \\ & & -4 & -10 & -4 \\ \hline & 2 & 5 & 2 & 0 \end{array}$$

$\therefore f(x) = (x-1)(x+2)(2x^2 + 5x + 2)$
$= (x-1)(x+2)(2x+1)(x+2)$

$\therefore f(x)$ has zeros at $x = 1, -2$ (order 2), $-\frac{1}{2}$



We do not know the exact locations of turning points in this region