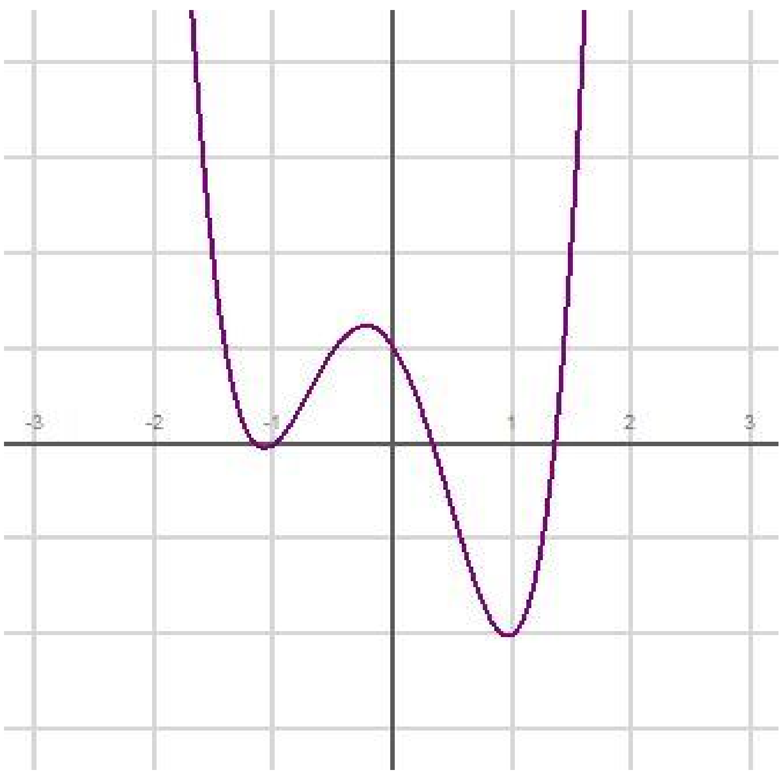
[ADVANCED FUNCTIONS]

Chapter 3 (This material is based on Chapter 4 in your text)

COURSE NOTES

POLYNOMIAL EQUATIONS AND INEQUALITIES



Chapter 3 – Polynomial Equations and Inequalities

Contents with suggested problems from the Nelson Textbook (Chapter 4)

3.1 Solving Polynomial Equations

Before embarking on this wonderful journey, it seems to me that it would be prudent to make some (seemingly silly) opening statements.

Seemingly Silly Opening Statements

1) Polynomial equations ARE NOT polynomial functions!
2) Solving any equation MEANS finding a solution (if a solution exists)!
3) Solving a polynomial equation is ALWAYS equivalent to finding the zeros of some polynomial function!

Example 3.1.1 (***back to Grade 9***)

Solve the linear equation

$$3(x-5)+2=5x+6$$

Goal

"$x =$ answer"

$3x-15+2=5x+6$

$-2x=19$

$x=-\frac{19}{2}$ (unique soln)

Example 3.1.2 (***remember grade 11?***)

Solve the quadratic equation

$$5x(x-1)+7=2x^2+9$$

$\boxed{\text{Goal}}$ (Two steps for poly. eqns of order 2 or more)

① Get the eqn in std form

"polynomial $= 0$"

①b do some work

② "$x =$ answer"

$5x^2-5x+7=2x^2+9$

$3x^2-5x-2=0$

$3x^2-6x+x-2=0$

x	+
-6	-5

-6, +1

$3x(x-2)+1(x-2)=0$

$\Rightarrow (x-2)(3x+1)=0$

$x=2$ or $x=-\frac{1}{3}$

①b "some work"

① Factor if possible

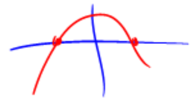
② Graphing Tech.

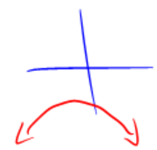
Geometrically speaking, solving a quadratic **equation** is equivalent to finding the zeros of a quadratic **function**.
Solving the **equation** in **Example 3.1.2** means the same thing as finding the zeros of the **function**

$$f(x) = 3x^2 - 5x - 2$$

Note further that quadratic **functions** can have

2 zeros **1 zero** **0 zeros**





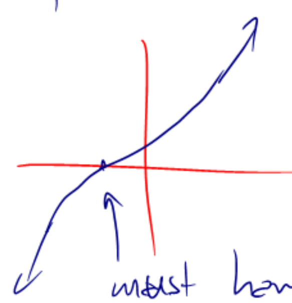
Thus quadratic **equations** can have **2 solutions**, **1 solution** or **no solutions**!

Comments about Higher Order Polynomial Equations

Consider the cubic **EQUATION** $x^3 + 2x^2 - 5 + 1 = 0$.
Q. How many ~~zeros~~ Solutions/roots can this equation have?
Ans.

1, 2, 3

Consider the fn $f(x) = x^3 + 2x^2 - 5x + 1$



must have a zero

⇒ the eqn must have at least one soln

Consider the quartic equation $4x^4 - 3x^3 + 5x^2 - 3x + 1 = 0$.
Q. How many ~~zeros~~ solutions/roots can this equation have?
Ans.

0, 1, 2, 3, 4

Note: Solving Polynomial Equations requires writing the equation in **Standard Form**, which is: "polynomial = 0"

Example 3.1.3

Solve the polynomial equation by factoring:

$$4x^3 - 3x = 1$$

$$4x^3 - 3x - 1 = 0$$

T.V. ± 1

Let $f(x) = 4x^3 - 3x - 1$

$f(1) = 0 \Rightarrow (x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

write the original eqn in factored form.

$$(x-1)(4x^2+4x+1) = 0$$

$$\Rightarrow (x-1)(2x+1)^2 = 0$$

$\therefore x = 1$ or $x = -\frac{1}{2}$

WHOA - rational!

Example 3.1.4

Solve the equation by factoring:

$$12x^4 + 16x^3 - 11x = 13x^2 - 6$$

$$12x^4 + 16x^3 - 13x^2 - 11x + 6 = 0$$

Let $f(x) = 12x^4 + 16x^3 - 13x^2 - 11x + 6$

T.V $\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = 0$

$$\begin{array}{r|rrrrr} -1 & 12 & 16 & -13 & -11 & 6 \\ & & -12 & -4 & 17 & -6 \\ \hline & 12 & 4 & -17 & 6 & 0 \end{array}$$

deal w/ the cubic

$$\therefore (x+1)(12x^3 + 4x^2 - 17x + 6) = 0$$

Let $g(x) = 12x^3 + 4x^2 - 17x + 6$

T.V. $\pm 1, \pm 2, \pm 3, \pm 6$

As it turns out $g(\pm 1) \neq 0,\ g(\pm 2) \neq 0,\ g(\pm 3) \neq 0,\ g(\pm 6) \neq 0$

MAYBE there is a rational zero!!

Rational Zero Test

Consider $12x^3 + 4x^2 - 17x + 6 = 0$.

$\pm 1, \pm 2, \pm 3, \pm 4$
$\pm 6, \pm 12$

We now, when using the factor theorem, will "test for zeros" using 2 steps:

1) Test for integer zeros using factors of the constant term.

   $\pm 1, \pm 2, \pm 3, \pm 6$

2) Test for rational zeros, where we consider $x = \frac{b}{a}$

   a factor of the constant term

   a factor of the leading coefficient.

The possible rational zeros are:

$\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm\frac{2}{3}, \pm\frac{3}{2}, \pm\frac{3}{4},$

Back to **Example 3.1.4**

For $g(x) = 12x^3 + 4x^2 - 17x + 6$ the possible rational zeros are:

$\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm\frac{2}{3}, \pm\frac{3}{2}, \pm\frac{3}{4}$

$g(\frac{1}{2}) = 0$

$$\begin{array}{c|cccc} \frac{1}{2} & 12 & 4 & -17 & 6 \\ & & 6 & 5 & -6 \\ \hline & 12 & 10 & -12 & 0 \\ \div 2 & 6 & 5 & -6 & \end{array}$$

$\therefore (x-1)(2x-1)(6x^2-5x-6) = 0$

$\Rightarrow (x-1)(2x-1)(3x+2)(2x-3) = 0$

$\therefore x = 1, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}$

Example 3.1.5

Solve the equation $3x^3 - 4x + 2 = 0$.

Let $f(x) = 3x^3 - 4x + 2$

TV

integer $\pm 1, \pm 2$

rational $\pm \frac{1}{3}, \pm \frac{2}{3}$

TWO KINDS NOW!

As it turns out

$f(\pm 1) \neq 0$, $f(\pm 2) \neq 0$, $f(\pm \frac{1}{3}) \neq 0$

$f(\pm \frac{2}{3}) \neq 0$

$\therefore$ There are no factor (ie the equation d.n.f.)

But CUBIC EQUATIONS MUST have at least one soln

$\Rightarrow$ We turn to graphing tech.

$\therefore$ By graphing calc $x = -1.35$

Class/Homework for Section 3.1

Pg. 204 – 206 #1, 2, 6 – 8, 10 – 12, 14, 15
Note: for #14a) you may need to ask about "domain restrictions"