3.2 Linear Inequalities

Once again, it seems a good idea to begin with a couple of opening statements.

Absolutely Non-Silly Opening Statements

1) The **algebra** of inequalities is the **SAME** as the algebra on equality (i.e. solving equations), with two exceptions:

   a) If you MULTIPLY OR DIVIDE across an inequality by a **negative**, then reverse the direction of the inequality.

   b) We can have 2 sided inequalities – e.g. $3 - x \leq 5 + 2x < 3$

2) The **Solution Set** of inequalities is infinite. The set is a segment of the real number line

Example 3.2.1

Solve the (linear) inequality $3x - 2 > 4$.

$3x > 6$

$x > 2$

Sketch the soln set

The solution set is the set of numbers which makes the statement true

Example 3.2.2

Solve the two sided inequality $-2 > -4x + 5 \geq -3$.

subtract 5 everywhere

$\Rightarrow \quad -2-5 > -4x \geq -3-5$

÷ −4 everywhere

$\Rightarrow \quad -7 > -4x \geq -8$

$\frac{7}{4} < x \leq 2$

Solution Set

Example 3.2.3

Solve $5 \leq 3(x-2) - 4(x+3) \leq 12$

$5 \leq 3x - 6 - 4x - 12 \leq 12$

$5 \leq -x - 18 \leq 12$

$23 \leq -x \leq 30$

$-23 \geqslant x \geqslant -30$

Solution Set

Example 3.2.4

Write the following sketch of a solution set in interval and set notation:

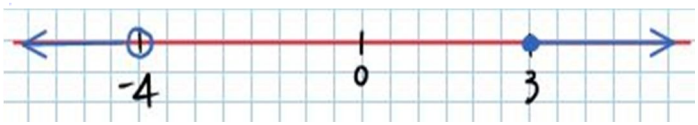


Figure 3.2.4

Interval $(-\infty, -4) \cup [3, \infty)$

Set Notation $\{x \in \mathbb{R} \mid -4 > x \geqslant 3\}$

$x < -4$, or $x \geqslant 3$

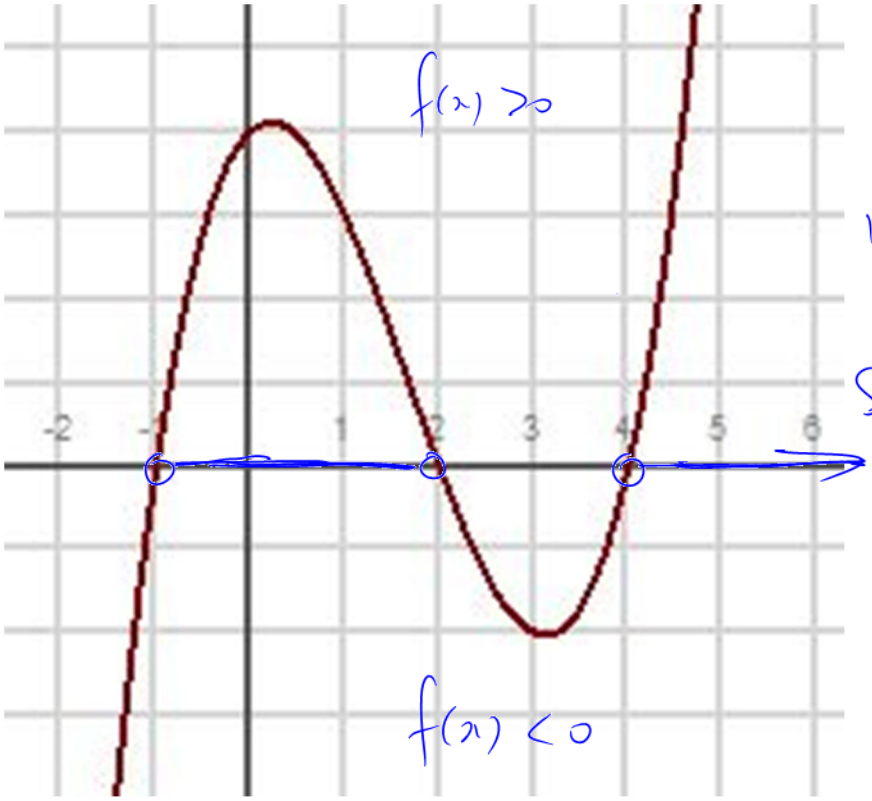
Graphical Views of (non-linear) Polynomial Inequalities

(the Algebra is tough…)

Example 3.2.5

Consider the sketch of the graph of some mystery cubic function.

Q. When (or better WHERE) are *the functional values* positive?



f(x) > 0 on

Interval: $(-1,2) \cup (4,\infty)$

Set: $\{x \in \mathbb{R} \mid -1 < x < 2,\ x > 4\}$

Figure 3.2.5

Example 3.2.6

Consider the sketch of the quartic $g(x)$, and determine where

a) $g(x) \le 0$

b) $g(x) > 2$

c) $-1 \le g(x) \le 2$

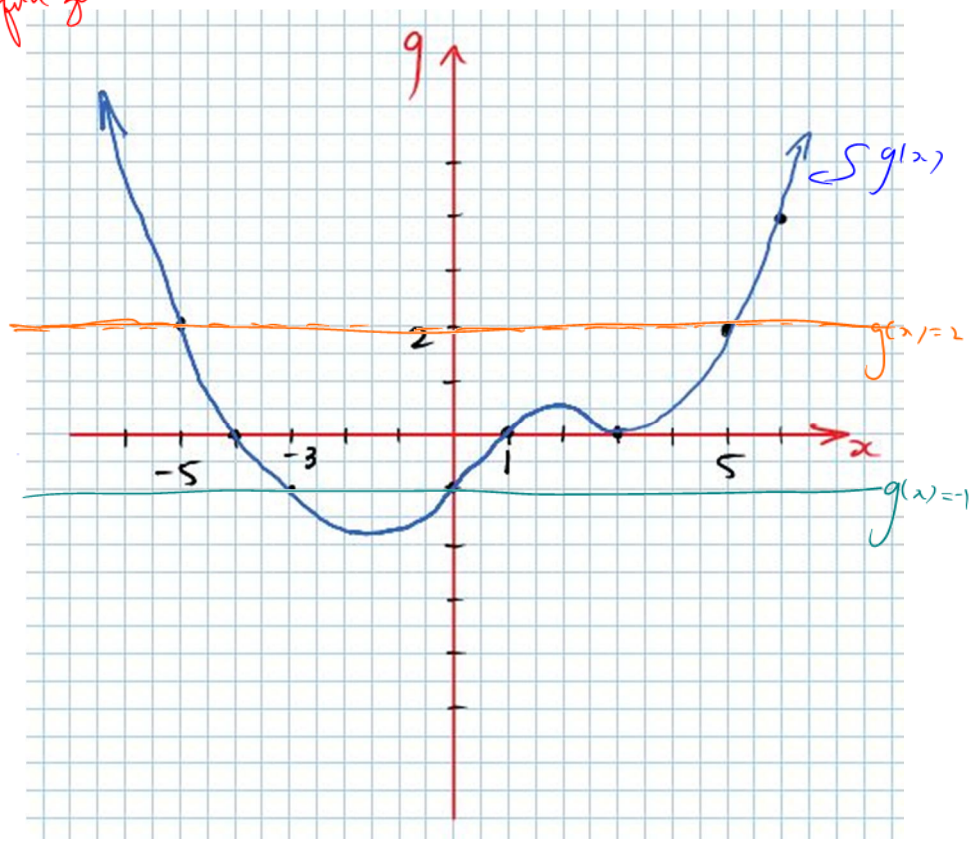


Figure 3.2.6

Class/Homework for Section 3.2

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