peter park
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3.3 Solving Polynomial Inequalities

For this section, no opening statements are required….

Non-Required Opening Statement

Solving non-linear polynomial inequalities can be accomplished in two ways:

1) Graphically (sometimes called Geometrically)
2) Algebraically (which tends to be more useful)

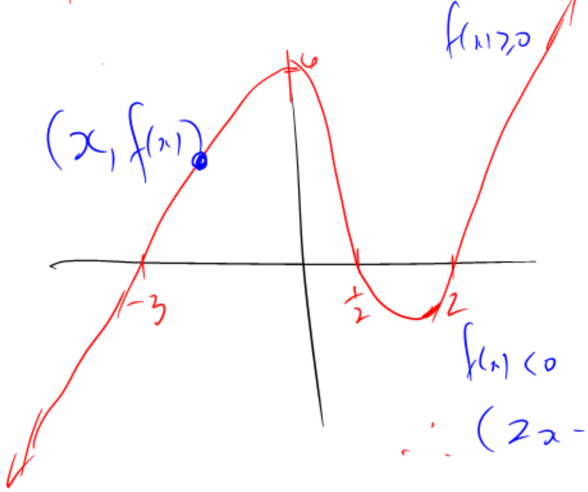
Example 3.3.1

Solve $(2x-1)(x-2)(x+3) \geq 0$.   **REMEMBER**: *FACTORED FORM IS YOUR FRIEND*

Graphically:

Let $f(x) = (2x-1)(x-2)(x+3)$

y int: $f(0) = 6$



Note: Solving an inequality graphically is rather easy, **BUT**

algebraically may require a lot of work

$\therefore (2x-1)(x-2)(x+3) \geq 0$ on

$[-3, \tfrac{1}{2}] \cup [2, \infty)$

Example 3.3.1 (Continued)

Solve $(2x-1)(x-2)(x+3) \geq 0$

Algebraically

For this technique we will construct an "**Interval Chart**", which can also be thought of as a "**table of signs**" (and wonders?)

Note: It is often helpful to remember that in mathematics we are dealing with **NUMBERS**.

Numbers have signs: Positive or Negative

e.g. $(x-2)$ is a **NUMBER** whose sign switches from +'ve to –'ve at $x=2$ (*i.e. the sign switches at the zero of the factor*)

The Interval Chart looks like:

Intervals	Split the Domain $(-\infty,\infty)$ at all **ZEROS** of the Factors		
Test Values	Choose a Domain	value inside each	Interval
Sign on 1st Factor			
Sign on 2nd Factor			
Sign on 3rd Factor			
Sign on the Product of Factors	Find the Intervals	with the **sign** we	want to answer the question

For our problem above, our chart will look like: $(2x-1)(x-2)(x+3) \geq 0$

INTERVALS	$(-\infty,-3)$	$(-3,\frac{1}{2})$	$(\frac{1}{2},2)$	$(2,\infty)$
TEST VALUES	-4	0	1	3
$x+3$	−ve	+ve	+ve	+ve
$2x-1$	−ve	−ve	+ve	+ve
$x-2$	−ve	−ve	−ve	+ve
product	−ve	+ve	−ve	+ve

$\therefore (2x-1)(x-2)(x+3) \geq 0$ on $[-3,\frac{1}{2}] \cup [2,\infty)$

Example 3.3.2

Solve algebraically $4x^4 + 16x^3 + x^2 - 39x - 18 < 0$.

Wait a second….where is your friend and mine…

FACTORED FORM

Let $f(x) = 4x^4 + 16x^3 + x^2 - 39x - 18$

$f(-2) = 0$

T.U.

INTEGER : $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

RATIONAL FACTORS 4 $(\pm 1, \pm 2, \pm 4)$

$\pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{9}{2}, \pm \frac{9}{4}$

-2	4	16	1	-39	-18
		-8	-16	30	18
	4	8	-15	-9	0

$\therefore$ we have

$(x+2)(4x^3 + 8x^2 - 15x - 9) < 0$

Let $g(x) = 4x^3 + 8x^2 - 15x - 9$

$g(-3) = 0$

T.U.

$\pm 1, \pm 3, \pm 9$ integer

rats : $\pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{9}{2}, \pm \frac{9}{4}$

-3	4	8	-15	-9
		-12	12	9
	4	-4	-3	0

$\therefore$ we have $(x+2)(x+3)(4x^2 - 4x - 3) < 0$

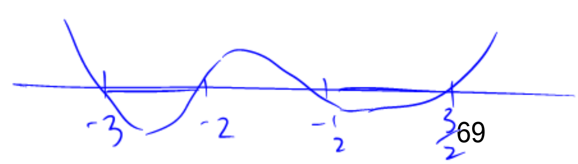
$4x^2 - 6x + 2x - 3$

$= 2x(2x-3) + 1(2x-3)$

$= (2x-3)(2x+1)$

$\Rightarrow (x+2)(x+3)(2x-3)(2x+1) < 0$

interval chart



INTERVALS	$(-\infty, -3)$	$(-3, -2)$	$(-2, -\frac{1}{2})$	$(-\frac{1}{2}, \frac{3}{2})$	$(\frac{3}{2}, \infty)$
TEST VALUES	-4	-2.5	-1	0	2
$x+3$	−ve	+ve	+ve	+ve	+ve
$x+2$	−ve	−ve	+ve	+ve	+ve
$2x+1$	−ve	−ve	−ve	+ve	+ve
$2x-3$	−ve	−ve	−ve	−ve	+ve
product	+ve	−ve	+ve	−ve	+ve

$\therefore\ 4x^4 + 16x^3 + x^2 - 39x - 18 < 0$ on

$$(-3, -2) \cup \left(-\frac{1}{2}, \frac{3}{2}\right)$$

Class/Homework for Section 3.3

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