4.2 Sketches of Graphs of Rational Functions

Recall the graph of $f(x)$ is

$$\{(x, f(x)) \mid x \in D_f\}$$

Note: In Advanced Functions we will only consider rational functions of the form

$$f(x)=\frac{ax+b}{cx+d} \qquad \frac{\text{order } 1}{\text{order}}$$

$ax+b=0 \Rightarrow x=-\frac{b}{a}$

Rational Functions of the form $f(x)=\frac{ax+b}{cx+d}$ will have:

1) One Vertical Asymptote

denominator $\neq 0$

$$x=-\frac{d}{c}$$

(eqn) $\frac{a}{b}=0 \Rightarrow a=0$

2) One Zero (unless $a=0$)

$$x=-\frac{b}{a}$$

(value)

3) Functional Intercept

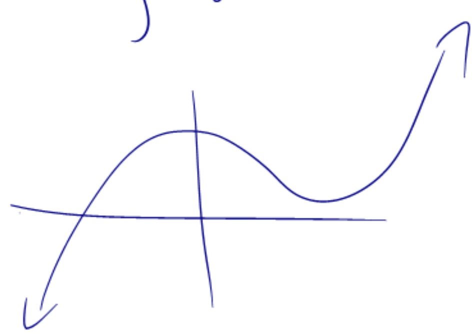
y-int $\quad f(0)=\frac{b}{d}$

4) A Horizontal Asymptote

$$y=\frac{a}{c}$$

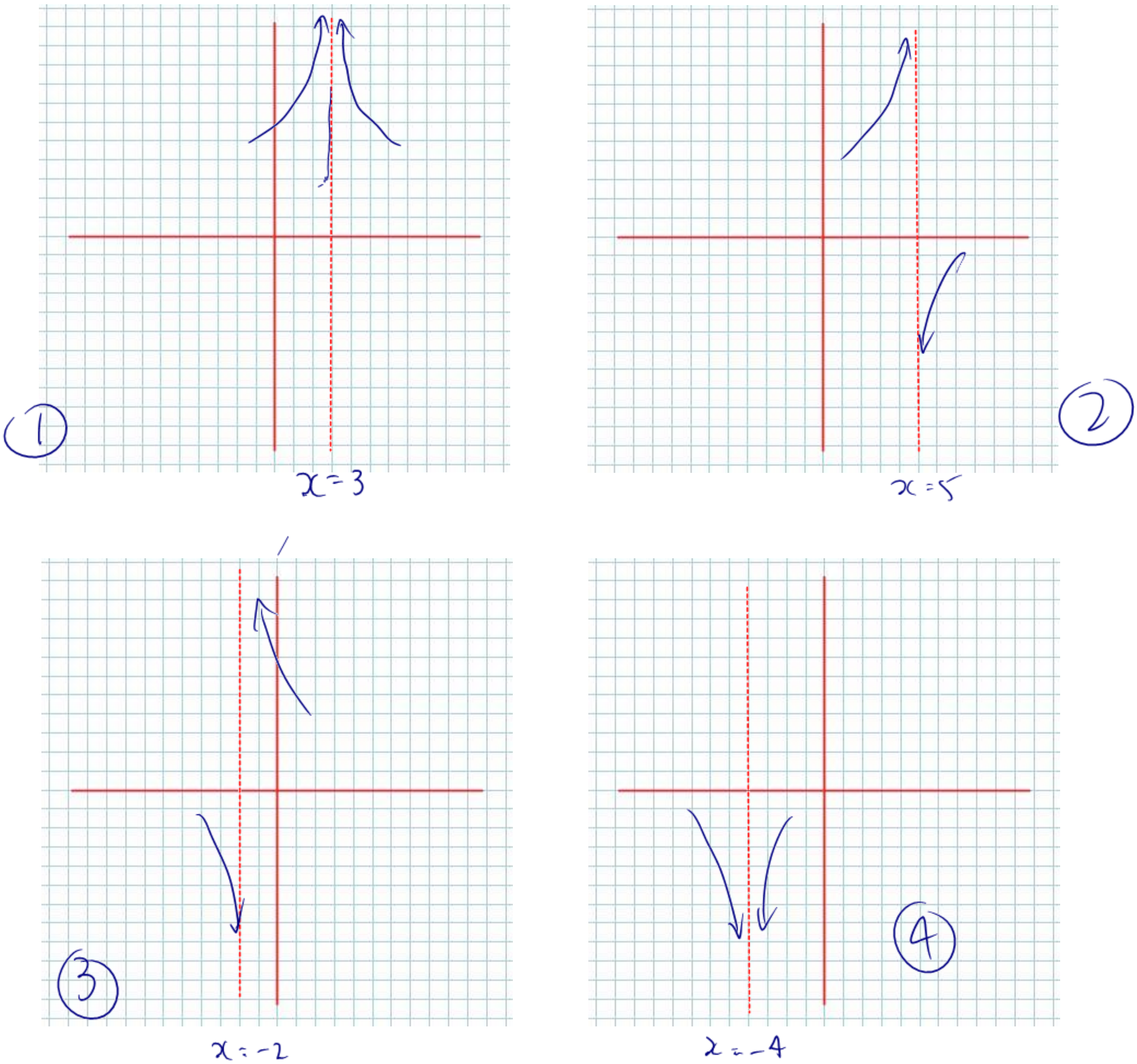
5) These functions will always be either always increasing or

always decreasing



Functional Behaviour Near A Vertical Asymptote

There are **FOUR** possible functional behaviours near a V.A.:



For functions of the form $f(x)=\dfrac{ax+b}{cx+d}$ we will see behaviours 2 or 3

The questions is, **how do we know which**?

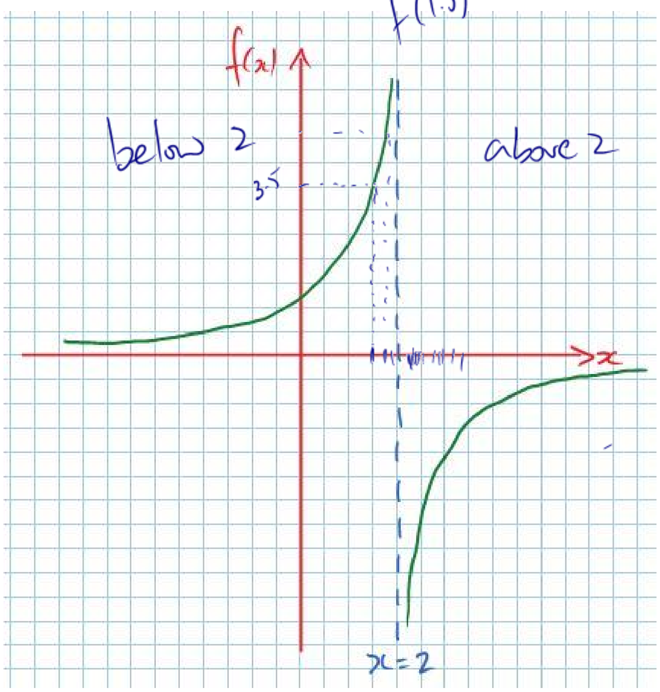
We need to **analyze** the function **near the V.A.**

→ using mathematics

We need to become familiar with some **Notation.**

Consider some rational function with a sketch of its graph which looks like:

$f(1.5) = 3.5$



V.A. splits the Cartesian plane into 2 halves

from below

$$\lim_{x \to 2^-} \left(f(x) \right) = +\infty$$

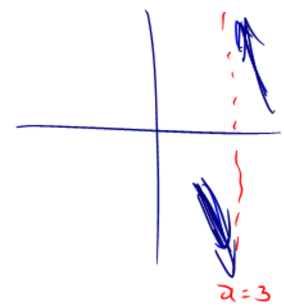
approaches but never equals 2

$$\lim_{x \to 2^+} \left(f(x) \right) = -\infty$$

from above

Example 4.2.1

Determine the functional behaviour of $f(x) = \dfrac{2x+1}{x-3}$ near its V.A.



V.A. $x = 3$

Consider

$$\lim_{x \to 3^-} \left(\frac{2x+1}{x-3} \right) = \frac{+7}{\text{"}-0\text{"}} = -\infty$$

$$\lim_{x \to 3^+} \left(\frac{2x+1}{x-3} \right) = \frac{7}{\text{"}+0\text{"}} = +\infty$$

We now have the tools to sketch some graphs!

Example 4.2.2

Sketch the graph of the given function. State the domain, range, intervals of increase/decrease and where the function is positive and negative.

a) $f(x)=\dfrac{2x+1}{x-1}$

V.A. $x=1$

H.A. $y=2$

zero: $\left(-\frac{1}{2},\ 0\right)$

y_{int} $(0,-1)$

$D_f:\{x\in\mathbb{R}\mid x\neq 1\}$ $(-\infty,1)\cup(1,\infty)$

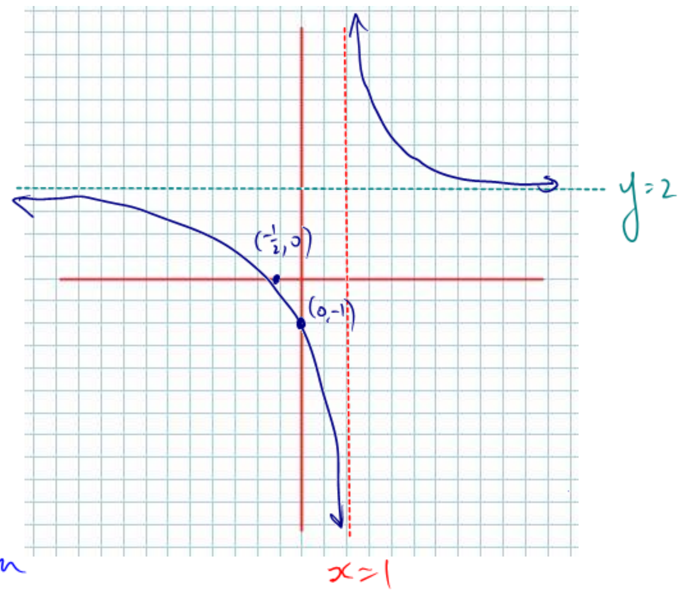
$R_f:\{f(x)\in\mathbb{R}\mid f(x)\neq 2\}$

$f(x)$ is increasing NEVER

$f(x)$ is decreasing on $x\in(-\infty,1)\cup(1,\infty)$

$f(x)>0$ on $x\in\left(-\infty,-\frac{1}{2}\right)\cup(1,\infty)$

$f(x)<0$ on $x\in\left(-\frac{1}{2},1\right)$



b) $g(x)=\dfrac{3x-2}{2x+5}$

V.A. $x=-\frac{5}{2}$

HA $y=\frac{3}{2}$

zero: $\left(\frac{2}{3},\ 0\right)$

y_{int}: $\left(0,\ -\frac{2}{5}\right)$

$D_g:\left\{x\in\mathbb{R}\mid x\neq -\frac{5}{2}\right\}$

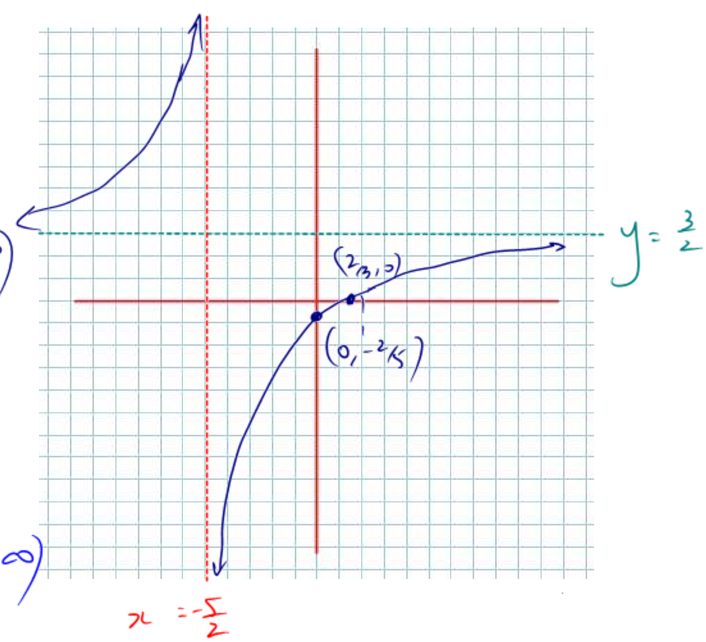
$R_g=\left\{g(x)\in\mathbb{R}\mid g(x)\neq\frac{3}{2}\right\}$

$g(x)$ is increasing on $x\in\left(-\infty,-5/2\right)\cup\left(-5/2,\infty\right)$

$g(x)$ is decreasing never

$g(x)>0$ on $x\in\left(-\infty,-5/2\right)\cup\left(2/3,\infty\right)$

$g(x)<0$ on $x\in\left(-5/2,\ 2/3\right)$



Example 4.2.3

Consider question #9 on page 274:

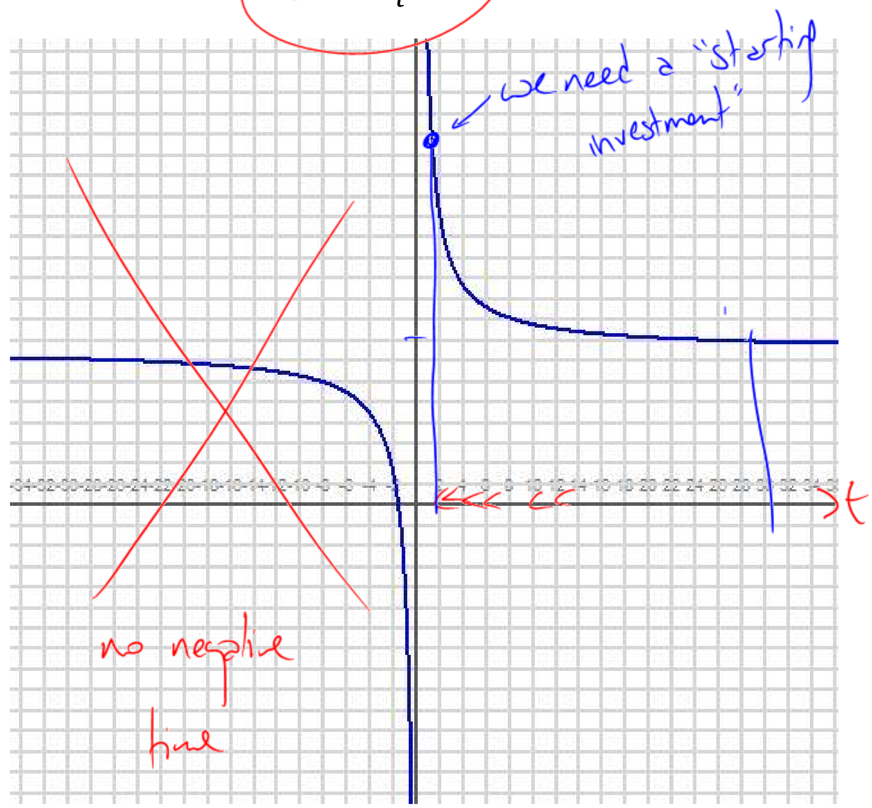
$$I(t) = \frac{15t + 25}{t}$$

This is a real world problem

I(t) is a mathematical MODEL

$∞ does not exist

The model cannot be trusted over its entire domain to represent reality. But we restrict the domain so that (for a "time") the model can be trusted



Class/Homework for Section 4.2

Pg. 272 #1, 2 (Don't use any tables of values!), 4 – 6, 9, 10