4.4 Solving Rational Equations

Solving a Rational Equation is **VERY MUCH** like solving a Polynomial Equation. Thus, this stuff is so much fun it should be illegal. But it isn't illegal unless you break a rule of algebra. Math Safe!

> **KEY** (*this is a major key for you music buffs*)
> **Multiplying by the Multiplicative Inverse of the Common Denominator** is wonderful to use **WHEN YOU HAVE** something like:
> $\text{RATIONAL}_1 + \text{RATIONAL}_2 = \text{RATIONAL}_3$

$\frac{3}{4} \cdot \frac{4}{3}$

e.g. $\frac{3}{x-3} = \frac{4(x+5)}{x} + \frac{3}{2}$

C.D. shuft $2x(x-3)$ FACTORED FORM

× everything by $2x(x-3)$

$2x(x-3)\left(\frac{3}{x-3}\right) = 2x(x-3)\left(\frac{4(x+5)}{x} + \frac{3}{2}\right)$

$\frac{6x(x-3)}{x-3} = \frac{8x(x-3)(x+5)}{x} + \frac{6x(x-3)}{2}$

$6x = 8(x-3)(x+5) + 3x(x-3)$ etc.

Make Sure To Keep RESTRICTIONS ON X In Mind

$x \neq 3, 0$

Example 4.4.1

rational = rational ⇒ cross multiply

a) Solve $\frac{x}{5}=\frac{9}{18}$

$\Rightarrow \frac{x}{5} = \frac{1}{2}$

$2x = 5$

$x = \frac{5}{2}$

b) Solve $\frac{1}{x}-\frac{5x}{3}=\frac{2}{5}$

RESTRICTIONS

$x \neq 0$

common denom $15x$

$15x\left(\frac{1}{x} - \frac{5x}{3}\right) = 15x\left(\frac{2}{5}\right)$

$\Rightarrow 15 - 25x^2 = 6x$

quadratic eqn

$25x^2 + 6x - 15 = 0$

$a = b$

$b = a$

dnf

Q.F. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = -0.90$ or $x = 0.66$

State the solns

c) Solve $\frac{3}{x}+\frac{4}{x+1}=2$

RESTRICTIONS

$x \neq 0,\ x \neq -1$

C.D. $x(x+1)$

$x(x+1)\left(\frac{3}{x}+\frac{4}{x+1}\right) = x(x+1)(2)$

$3(x+1) + 4x = 2x^2 + 2x$

$7x + 3 = 2x^2 + 2x$

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$\therefore x = -\frac{1}{2},\ x = 3$

d) Solve $\frac{10}{x^2-2x}+\frac{4}{x}=\frac{5}{x-2}$

FACTOR!!!!!

RESTRICTIONS

$x \neq 2,\ x \neq 0$

C.D. $x(x-2)$

$\frac{10}{x(x-2)} + \frac{4}{x} = \frac{5}{x-2}$

$x(x-2)\left(\frac{10}{x(x-2)}+\frac{4}{x}\right) = x(x-2)\left(\frac{5}{x-2}\right)$

$10 + 4(x-2) = 5x$

$10 + 4x - 8 = 5x$

$-x = -2$

$\Rightarrow x = 2$

BUT $x \neq 2$ by restrictions

$\therefore$ No soln.

e) Solve $16x-\frac{5}{x+2}=\frac{15}{x-2}-\frac{60}{(x-2)(x+2)}$

Restrictions

$x \neq 2,\ x \neq -2$

CD: $(x-2)(x+2)$

$(x-2)(x+2)\left(16x-\frac{5}{x+2}\right)=(x-2)(x+2)\left(\frac{15}{x-2}-\frac{60}{(x-2)(x+2)}\right)$

$16x(x-2)(x+2)-5(x-2)=15(x+2)-60$

$(16x^2-32x)(x+2)-5x+10=15x+30-60$

$16x^3+32x^2-32x^2-64x-5x+10=15x-30$

$16x^3-84x+40=0$

$\div 4$

$4x^3-21x+10=0$

Let $f(x)=4x^3-21x+10$

$f(2)=0$

TV.

$\pm 1, \pm 2, \pm 5, \pm 10$ int.

$\pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{5}{2}, \pm\frac{5}{4}$ rats

2	4	0	−21	10
		8	16	−10
	4	8	−5	0

$\therefore (x-2)(4x^2+8x-5)=0$

$(x-2)(2x-1)(2x+5)=0$

$\therefore x=2, \frac{1}{2}, -\frac{5}{2}$

inadmissable

but $x \neq 2$

$\therefore$ solns are $x=\frac{1}{2},\ x=-\frac{5}{2}$

Example 4.4.2

From your Text: Pg. 285 #10

rational!!!

This is about "rates" of production

"shift / time"

rate A + rate B = rate (together)

$\frac{\text{cases}}{\text{min}}$ A + $\frac{\text{cases}}{\text{min}}$ B = $\frac{\text{cases}}{\text{min}}$ (together)

$$\frac{1}{s} + \frac{1}{s+10} = \frac{1}{15}$$

restrictions

$s \neq 0, -10$

CD $15s(s+10)$

$$\Rightarrow 15s(s+10)\left(\frac{1}{s} + \frac{1}{s+10}\right) = 15s(s+10)\left(\frac{1}{15}\right)$$

$$15(s+10) + 15s = s(s+10)$$

$$\Rightarrow 15s + 150 + 15s = s^2 + 10s$$

$$s^2 - 20s - 150 = 0$$

By Q.F.

dnf.

$s = 25.8$ min

∴ A takes 25.8 min

B takes 35.8 min

Class/Homework for Section 4.4

Pg. 285 - 287 #2, 5 – 7def, 9, 12, 13