4.5 Solving Rational Inequalities

The joy, wonder and peace these bring is really quite amazing

e.g. Solve $\frac{x-2}{7} \geq 0$

3 expressions for the soln

$\times 7$ $\quad x-2 \geq 0$

$x \geq 2$

or

or $\frac{x-2}{7} \geq 0$ on $x \in [2, \infty)$

Note: For Rational Inequalities, with a variable in the denominator, you **CANNOT** multiply by the multiplicative inverse of the common denominator!!!!

Why? The denominator has a sign, and could be negative

$\Rightarrow$ we ~~might~~ have to reverse the direction of inequality

Example 4.5.1

Solve $\frac{x-2}{x+3} \geq 0$

We solve by using an Interval Chart

For the intervals, we split $(-\infty, \infty)$ at all zeros (**where the numerator is zero**), and all restrictions (**where the denominator is zero**) of the (SINGLE) rational expression. Keep in mind that it may take a good deal of algebraic manipulation to get a SINGLE rational expression…

Ex 1: $\frac{x-2}{x+3} \geq 0$ $\quad$ restrictions: $x \neq -3$

zeros: $x = 2$

Intervals	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
T.V.	-4	0	3
$x-2$	$-ve$	$-ve$	$+ve$
$x+3$	$-ve$	$+ve$	$+ve$
$\frac{x-2}{x+3}$	$+ve$	$-ve$	$+ve$

Signs

not] since $x \neq -3$

include 2 since it's not a restriction

$\therefore \frac{x-2}{x+3} \geq 0$ on $x \in (-\infty, -3) \cup [2, \infty)$

Example 4.5.2

Solve $\frac{1}{x+5} < 5$

DO NOT CROSS MULTIPLY or else

- Get everything on one side
- Simplify into a single Rational Expression using a common denominator
- ~~Interval Chart it up~~ simplify numerator

$$\Rightarrow \quad \frac{1}{x+5} - \frac{5}{1} < 0$$

$$\Rightarrow \quad \frac{1}{x+5} - \frac{5(x+5)}{x+5} < 0$$

$$\Rightarrow \quad \frac{1-5(x+5)}{x+5} < 0$$

$$\Rightarrow \quad \frac{-5x-24}{x+5} < 0$$

zeros : $x = -\frac{24}{5}$

restriction : $x \neq -5$

Intervals	$(-\infty, -5)$	$(-5, -\frac{24}{5})$ -4.8	$(-\frac{24}{5}, \infty)$
TV	-6	-4.9	0
$-5x-24$	+ve	+ve	−ve
$x+5$	−ve	+ve	+ve
$\frac{-5x-24}{x+5}$	−ve	+ve	−ve

$\therefore \quad \frac{1}{x+5} < 5$ on $x \in (-\infty, -5) \cup (-\frac{24}{5}, \infty)$

Example 4.5.3

Solve $\dfrac{x^2+3x+2}{x^2-16} \geq 0$

FACTORED FORM IS YOUR FRIEND

we need zeros and restrictions!

$$\Rightarrow \quad \frac{(x+2)(x+1)}{(x-4)(x+4)} \geq 0$$

zeros: $x = -2, -1$

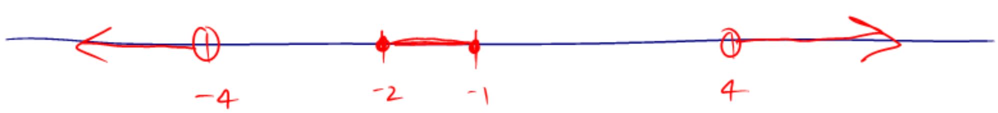
restrictions $x \neq 4, -4$

Intervals	$(-\infty, -4)$	$(-4, -2)$	$(-2, -1)$	$(-1, 4)$	$(4, \infty)$
T.V.	-5	-3	-1.5	0	5
$x+2$	−ve	−ve	+ve	+ve	+ve
$x+1$	−ve	−ve	−ve	+ve	+ve
$x-4$	−ve	−ve	−ve	−ve	+ve
$x+4$	−ve	+ve	+ve	+ve	+ve
$\frac{(x+2)(x+1)}{(x-4)(x+4)}$	+ve	−ve	+ve	−ve	+ve

be careful w/ brackets

$$\therefore \quad \frac{x^2+3x+2}{x^2-16} \geq 0 \quad \text{on} \quad x \in (-\infty, -4) \cup [-2, -1] \cup (4, \infty)$$

Solution set sketch



Example 4.5.4

Solve $\frac{3}{x+2} \le x$

$$\Rightarrow \quad \frac{3}{x+2} - x \le 0$$

$$\Rightarrow \quad \frac{3 - x(x+2)}{x+2} \le 0$$

$$\Rightarrow \quad \frac{-x^2 - 2x + 3}{x+2} \le 0$$

× both sides by -1

$$\frac{x^2 + 2x - 3}{x+2} \geq 0$$

⇐ eliminates the "−" on x^2

$$\Rightarrow \quad \frac{(x+3)(x-1)}{x+2} \geq 0$$

zeros : $x = -3$, $x = 1$

restrictions : $x \neq -2$

Intervals	$(-\infty, -3)$	$(-3, -2)$	$(-2, 1)$	$(1, \infty)$
T.V.	-4	-2.5	0	2
$x+3$	−ve	+ve	+ve	+ve
$x-1$	−ve	−ve	−ve	+ve
$x+2$	−ve	−ve	+ve	+ve
$\frac{(x+3)(x-1)}{x+2}$	−ve	+ve	−ve	+ve

−2 is a restriction!

$$\therefore \quad \frac{3}{x+2} \le x \quad \text{on} \quad [-3, -2) \cup [1, \infty)$$

Example 4.5.5

From your Text: Pg. 296 #6a

Using **Graphing Tech**

Solve $\dfrac{x+3}{x-4} \geq \dfrac{x-1}{x+6}$

Note: There are **TWO** methods, both of which require a **FUNCTION** (let $f(x) = \ldots$ returns)

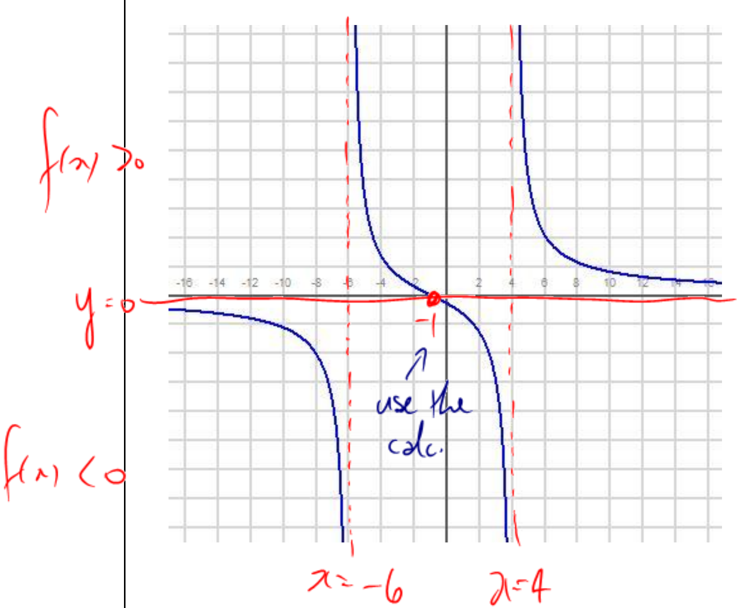
don't simplify calculator will do the work

Preferred Method

1) Get a Single Function (on one side of the inequality)

$\Rightarrow \dfrac{x+3}{x-4} - \dfrac{x-1}{x+6} \geq 0$

Let $f(x) = \dfrac{(x+3)}{(x-4)} - \dfrac{(x-1)}{(x+6)}$



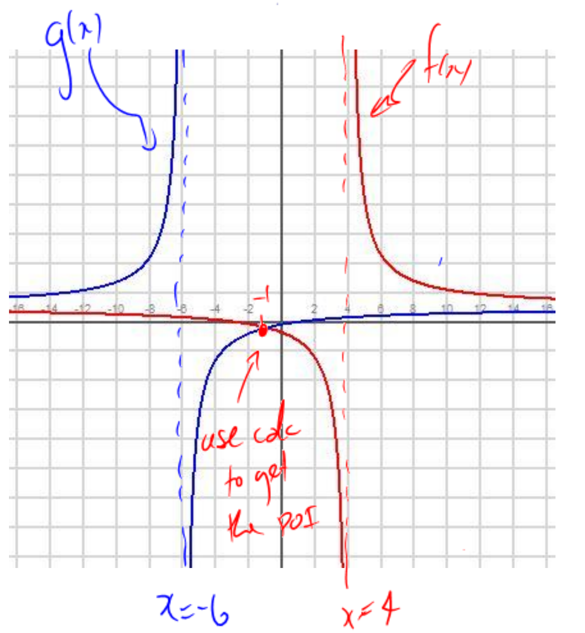
$\therefore \dfrac{x+3}{x-4} \geq \dfrac{x-1}{x+6}$ on $(-6, -1] \cup (4, \infty)$

2) Use Two Functions (one for each side)

Let $f(x) = \dfrac{x+3}{x-4}$

$g(x) = \dfrac{x-1}{x+6}$

$f(x) \geq g(x)$



$\dfrac{x+3}{x-4} \geq \dfrac{x-1}{x+6}$ on $(-6, -1] \cup (4, \infty)$

Class/Homework for Section 4.5

Pg. 295 - 297 #1, 3, 4 – 6 (def), 9, 11